Optimal Dynamic Production Policy: The Case of a Large Oil Field in Saudi Arabia*

Weiyu Gao
Department of Economics, Rice University
Houston, Texas
wgao68@hotmail.com

Peter Hartley
Department of Economics, Rice University
Houston, Texas
hartley@rice.edu

Robin C. Sickles
Department of Economics, Rice University
Houston, Texas
rsickles@rice.edu

February 20, 2004

Abstract

We model the optimal dynamic oil production decisions for a stylized oilfield resembling the largest developed light oil field in Saudi Arabia, Ghawar. We use data from a number of sources to estimate the cost and revenue functions used in the dynamic programming model. We also pay particular attention to the dynamic aspects of oil production. We use a nonparametric smoothing technique – tensor splines – to approximate the value function. The optimal solution depends on assumptions about various exogenous variables such as the discount rate and the timing of breakthroughs in the use of alternative energy, which we take to be solar energy. We account for uncertainty about the forecasts by examining the solutions under a number of scenarios. Our model is based on the hypothesis that oil production is chosen to maximize the discounted value of profits. Saudi oil policy reflects many political and strategic motives. Our analysis enables one to quantify the cost of pursuing these non-economic objectives.

JEL Classification System Number: C30, C61, Q32, Q41

Keywords: Optimal oil production; Dynamic programming; Value function approximation; Tensor splines.

*Funding for this study was provided by a grant from the Center for International Political Economy (CIPE) to the James A. Baker III Institute for Public Policy at Rice University. The authors would like to thank Dr. Stephen Rester of Haliburton Industries and international industry expert and private consultant Richard Martin of Houston for their vital contributions in data analysis and in providing modeling expertise that allowed us to simulate the temporal patterns of reservoir production used in our analysis. The usual caveat applies.

1. Introduction

Understanding the dynamics of the international oil market has been a major concern for economists for at least the last three decades. The importance of successfully modeling the international oil market and the related issue of OPEC decisions about oil production was brought home when the Arab oil embargo of 1973–74 resulted in a four-fold increase in the world oil price. Saudi Arabia, the largest supplier in OPEC, then as now produces close to 30% of total OPEC output and about 12% of the total world output. It also has roughly 30% of the world's proven oil reserves and a maximum sustainable production capacity of a little less than 10 million barrels per day. The overwhelming influence of Saudi oil policy on past world oil prices is widely acknowledged. Current oil price fluctuations only reinforce its dominant position in world oil pricing behavior.

Oil pricing and production decisions in Saudi Arabia are determined at the highest level of the government. Saudi government expenditure is heavily financed by oil revenues and oil-related activities such as petrochemicals and refining, which account for about 50% of Saudi government expenditures (Azzam, 1993). Saudi Arabia's oil policy also is motivated by more than maximization of profits or market share. Askari (1991) suggests that historically broad political goals and economic factors apart from profits have motivated their oil policy. The political concerns include Saudi's role in the world, Arab solidarity, and regional politics. Economic factors apart from profits include a desire for diversification away from oil revenue in the long term and a desire to meet short-term fluctuations in domestic and foreign expenditures.

It is difficult to discern exactly how these factors affect Saudi's oil production decisions and the world oil market. The approach we take is to focus on finding the dynamically optimal (profit maximizing) oil production rate from a homogeneous light oil field whose field properties mirror the giant Ghawar field. This field accounts for about 60%–70% of Saudi oil reserves. We restrict attention to a single field for a number of reasons. The major Saudi fields are very large and we need a complex reservoir description specific to each field. There is also a lack of individual well production history for many fields. Finally, the time and manpower needed, and computing resources required, for a national model would be large even if the data were available. Our analysis allows us to assess the potential for forgone profits in the event that oil production decisions are based on criteria other than the maximization of the expected present value of profits and provides a measure of the extent to which long-run value maximization is being followed.

A number of studies have addressed the oil production policies of OPEC, and Saudi Arabia in particular, in a dynamic setting. Dahl and Yucel (1991), among others, find empirical support for the role of dynamic behavior in producer supply decisions. Quandt (1982) provides a detailed qualitative discussion of Saudi Arabia's oil policy (in the 1970s and 1980s) and its possible motivations based on two criteria: optimization of the long-term value of oil reserves and the attainment of political goals. Khadduri (1996) discusses the oil policies of Middle East countries in the context of current developments in that region. Powell (1990) gives an excellent review of two major strands of economic research on OPEC oil production. The first approach attempts to simulate the behavior of the decision-maker. The second is the intertemporal optimization approach, traditionally attributed to Hotelling (1931). Recent research on oil production using dynamic value optimization has been pursued by, among others, Wirl (1990), Suranovic (1993), Benkherouf (1994), Lohrenz and Bailey (1995), and Fousekis and Stefanou (1996).

Our paper differs from previous work on these issues in that we consider not only the economic factors that affect Saudi Arabia's oil policy, like demand and costs, but also relevant reservoir engineering variables, such as water injection, the cumulative production of the field, and the number of oil wells. Using engineering computer models of dynamic fluid flow (*WorkBench* Black Oil Simulator, 1995), we simulate the effects of these reservoir conditions on the cost of production and the short-run production capacity in an oil field whose production characteristics mirror those of Saudi Arabia's largest light oil field, Ghawar. The engineering model of the production process recursively solves a system of homogeneous difference equations that describe the fluid dynamics within and among a set of three-dimensional grids that partition the oil field and whose joint behavior describes the production dynamics of the overall field. A short-term dynamic production function for the field is then estimated using the Black Oil simulation results. This dynamic production function is employed in the dynamic optimization model as an inequality constraint qualifying the intertemporal profit maximization problem.

Our approach also contrasts with the standard approach taken in the economics of production literature of specifying the production function based either on some generic property of scale or substitution (e.g., the constant elasticity of substitution production function), or on approximating forms (such as the translog and Leontief). It is related to the work of Griffin (1977, 1978) who constructed approximations to static technologies utilizing pseudo-data. However, it differs from that approach in that the production technology we consider is fundamentally dynamic.

The paper is organized as follows. Section 2 introduces and details our dynamic profit-maximizing model. Section 3 discusses data sources and the methodology used to estimate the dynamic programming model. This requires specification and estimation of revenue, cost, and dynamic production functions. Section 4 addresses theoretical issues regarding the existence and uniqueness of the optimal solutions. Proofs of the theorems are in the technical appendix. Simulations of the optimal solutions to our model under a number of scenarios are presented in section 5. Section 6 contains concluding remarks.

2. Dynamic Modeling of Oil Production Decisions[1]

In order to build a representative model for Saudi Arabia's oil production decisions, a number of issues need to be considered. First, the demand equation for Saudi Arabian oil should be studied based on the structure of the world oil market. Second, the oil production cost function needs to be correctly specified. Costs include exploration and development costs in addition to the costs of producing output from existing wells. Third, the dynamic nature of oil production needs to be addressed. Current oil production affects reservoir conditions and hence future production costs and capacity. Fourth, the optimization problem will not in general be stationary because the demand function for Saudi oil will vary over time. We approximate the optimal policy by postulating a stationary demand function beyond some period T in the future when the energy market is assumed to be dominated by a backstop technology (or technologies) and Saudi Arabia effectively becomes a price taker in the market for energy. Having solved this stationary "tail problem", we then calculate time-varying value functions in earlier periods using backwards recursion.

We attempt to keep our model as comprehensive as possible given the constraints we faced on how complex the model could be. This is detailed in the next section. Our research focuses on the production of oil from a homogeneous light oil field whose field properties mirror the giant Ghawar field that accounts for about 60%–70% of Saudi oil reserves. We recognize, however, that oil

[1]An exhaustive and complete model of optimal oil policy, particularly of a Middle East country like Saudi Arabia, not only involves specifying the interactions among the major players inside and outside OPEC, but also requires a thorough understanding of the country's specific economic and political situations. While acknowledging the complexity, we believe that there is much to be gained by focusing on oil profitability. In particular, even when the maximization of the expected present value of profits is not Saudi Arabia's primary goal, it is of interest to know how much other objectives cost in terms of foregone profits. Alternatives to dynamic profit maximization have been proposed in the literature. For example, Teece's (1982) model is based on the assumption that firms base production decisions on target revenues wherein price raises should lead to substantial reductions in OPEC supply, an empirical implication that does not appear to be consistent with OPEC behavior in the last decade or so.

fields in Saudi Arabia are not homogeneous in their reservoir environments. This could further strain the realism of our modeling paradigm.

We model the optimal production policy for our hypothetical light oil field using the following Bellman equation[2]

$$v_t\left(N_t, CP_t\right) = Max_{X_t, dN_t}\{r\left(X_t\right) - c\left(X_t, dN_t, W_t, N_t\right) + \beta v_{t+1}\left(N_{t+1}, CP_{t+1}\right)\} \quad (2.1)$$

subject to

$$N_{t+1} = N_t + dN_t$$

$$CP_{t+1} = CP_t + 365X_t$$

$$W_t = w\left(X_t, N_t, dN_t\right)$$

$$0 \le X_t \le f\left(N_t, dN_t, W_t, CP_t\right) \text{ and } dN \ge 0$$

where *X*, *dN*, *W*, *N*, and *CP* stand correspondingly for daily oil production rate (in millions of barrels), the number of new oil wells drilled during the period, water injected daily (in millions of barrels) to maintain the reservoir liquid pressure, the number of existing producing wells in the beginning of a period, and the cumulative production of the field. *X* and *dN* are policy variables and *N* and *CP* are state variables. β is the discount factor. *r* denotes revenue function, which can be formulated as *p(X)X*, where *p* stands for the inverse demand equation that relates the equilibrium world oil market price to Saudi's supply. Increasing output in an oil field often requires additional water injection, and this relation is specified here as function *w*. The function *f* represents the short-term field capacity that constitutes an upper bound for oil production during a certain period.

Powell (1990) points out that the solutions to intertemporal optimization models for OPEC countries will be strongly influenced by the future cost of oil substitutes in the terminal stationary environment and the discount rate(s) of the decision-makers. We simulate the optimal production paths based on four scenarios in order to examine the robustness of the solutions both to variations in the discount rate and to different assumptions about factors that affect the evolution of oil demand in the future, such as the timing of energy substitution. In all cases, the model is not assumed to be stationary before the substitute backstop technologies are available.

The first two simulation scenarios are associated with different discount rates. Our choices of discount rates (10% and 30%) are consistent with the results of Adelman (1993a) who discusses

[2]Under certain general and nonrestrictive conditions (2.1) can be shown to be equivalent to a sequence problem (SP). For these conditions, see Theorem 4.2 and 4.3 in Stokey, Lucas, and Prescott (1989).

oil-producing countries' discount factors. According to his research, 10% is a standard discount rate for countries like the United States and others that can diversify their income to a significant degree. On the other hand, for some OPEC countries, like Saudi Arabia, the discount rate may exceed 20% or even three times the standard rate. The reason is that the Saudi government relies heavily on oil income, which implies that there is a substantial risk associated with the income stream provided by exploiting oil resources.

The other two scenarios examine the effects on Saudi's long-term oil production policy of future cost reductions in one of the oil substitutes – solar energy. The difference between these two simulations lies in the timing of a breakthrough in producing solar energy on a massive scale. In the solar energy literature, there are different predictions of the likely cost reduction of solar energy production. Among them, Cody and Tiedje (1992) predict a fall in the cost of photovoltaic electricity from 40 cents per kWh to 7–12 cents per kWh by 2010. Dracker and De Laquil III (1996) forecast that the cost of electricity from solar energy would fall to 3.5–10.6 cents per kWh by 2005 –2010. Research conducted by the U. S. Department of Energy's National Center for Photovoltaics indicate that with intensified R&D, the cost of photovoltaics may fall below the cost of electricity from fossil fuel around 2026. We adopt relatively conservative expectations in our simulations. In one scenario, the cost of photovoltaic electricity starts to fall below that of electricity from fossil fuel in 2036, and in the other it starts in 2026. A ten-year transition period is assumed in both cases during which much of the demand for Saudi Arabian oil gradually switches to solar energy[3]. A detailed description of the transition scenario from oil to solar energy is presented in section 5.

3. Data and Estimations

The optimal control problem is based on the maximization of discounted net profits. This section outlines how we obtain functional representations for the required revenue and cost functions and the production process.

[3] Another issue that could be investigated in more detail is that different components of the energy market could abandon fossil fuel at different times. Fuel Cells or other technologies such as electric vehicles or the use of methanol as a fuel could also displace oil from transportation applications before solar energy does. The latter may only be feasible if improved batteries allow electric vehicles to compete with vehicles burning liquid fuel or we can eventually use hydrogen as a fuel.

3.1 The Revenue Function

The revenue function specifies price times the level of extraction as a function of the rate of extraction from the oil field. We base our revenue function on the Oil Market Simulation model (OMS) developed by the Energy Information Administration (EIA) of the United States. The OMS model, whose geographic coverage includes almost all major market economies, is an annual model that projects the world oil market to year 2010 from a database that begins in 1979. Net imports from the formerly centrally planned economies are taken as exogenous. The model estimates the effects of price changes on oil supply and demand, and computes an oil price path over time that allows supply and demand to remain in balance within the market economies as a whole. It has been extensively used in the EIA analyses since 1978 and has been used for the Agency's annual report to the U.S. Congress. The key assumption in the OMS model is that the current oil price, GDP growth rates, exchange rates, and the previous years' supplies and demands are the only determinants of the supply and the demand from non-OPEC and non-Communist countries. All oil exporting countries except those in OPEC are assumed to be price takers. OPEC is assumed to be a Stackelberg type of residual supplier.[45] Specifically, OPEC is assumed to set the market price by following a price-reaction function that increases price with increased capacity utilization, where capacity is defined as maximum sustainable production. The resulting OMS model consists of 12 equations (5 supply, 5 demand, OPEC production, and world price) and 2 identities. OMS projects a market clearing price for each year that equates world oil demand and the sum of oil production from all sources, including inventory changes. Using the OMS model, we simulate 25 different production (X_t) and price (P_t) trajectories over the simulation period (1986–2010) and the corresponding price and production responses from the residual supplier – the OPEC countries. Utilizing these OMS simulation results, we specify and estimate the following inverse demand equation that describes the relationship between the demand for Saudi oil and its corresponding pricing policy:

$$\log P_t = \alpha_0 + \alpha_1 X_t + \alpha_2 T + \varepsilon \qquad (3.1)$$

[4]Griffin and Neilson (1995) broaden the array of possible pricing strategies by OPEC considered in the Griffin (1985) study that found support for cartel behavior by OPEC. They find that strategic behavior may have changed in the mid-1980's from a dominant firm with strong cartel overtones to a tit-for-tat strategy (Salehi-Isfahani (1995, p. 16).

[5] Crϑmer, J., and D. Salehi-Isfahani (1989) provide a convincing theoretical argument that the oil market is competitive and that collusive or oligopolistic behavior is not necessary to explain the price fluctuations of the 1970's and 1980's. Rather, they argue that "…absorptive capacity constraints and imperfections in the international capital markets…" (p. 431) cause the supply curve of oil to be backward bending. Thus with relatively low demand elasticities there maybe

where T is the time trend index defined as: $T = t/60$ if $t \leq 60$ and $T = 1$ if $t > 60$, and t starts at 1 in year 1986 and equals to 60 in year 2045. This specification most likely simplifies some important details. We wish to keep the format relatively uncomplicated to facilitate calculation of the value function while allowing us to account for the monopolistic behavior of Saudi Arabia. The time index, T, is included to capture the effects of changing technologies, preferences and levels of development as well as other omitted variables that affect demand. The ordinary least squares regression results are reported in Table 3.1.

Table 3.1: Estimation of the Inverse Demand Equation

	α_0	α_1	α_2
Estimates	*3.5323*	*-0.0398*	*3.9656*
Standard Deviation	*0.0520*	*0.0023*	*0.1557*
R^2	*0.7476*		

These provide us with the annual revenue function for the field used in the non-stationary periods. (3.1) is re-expressed as

$$R(X_t) = (365X_t)\cdot e^{3.5323-0.0398(37X_t)+3.9656T} \quad (3.2)$$

3.2 The Production Cost Function

Studies of optimal oil production in OPEC countries typically do not include a detailed model of production cost. This is no doubt explained by the difficulty of obtaining accurate information. In general, oil production cost usually consists of three parts: (1) exploration cost, (2) development cost, and (3) operating costs. We use information from different sources to construct a complete cost function for the Saudi light oil field.

Exploration cost includes the cost incurred in geological and geophysical surveys for discovery of new fields, and the drilling of exploration wells. While we have data on the cost of drilling producing wells, the cost of an exploration well in Saudi light oil field is not available. Nor do we have sufficient data to obtain an exact form of exploration expenditure as a function of total oil production. However, according to Masseron (1990, p. 98), exploration expenditure accounts

two stable equilibria, one consistent with the low price era that existed between the other high price eras that followed the discontinuous price jumps of the oil crises.

for about 10–20% of the total production cost. Therefore, after estimating the development and production costs, we add 20% to account for exploration expenditure.

Development costs can be divided into two general categories: (1) infrastructure cost, i.e., surface installation and maintenance costs, and (2) oil well costs, which include investment for new wells and maintenance cost for old ones. Due to data limitations, only a weighted annual average infrastructure cost per barrel (bbl) is calculated for Arabian light fields (like Abu hadriyah, Ghawar, etc.), and Arabian medium fields (like Zuluf, Aatif, etc.), which together usually account for about 70% of total Saudi output. The weights used for calculating average surface cost are the relative production ratios of each type of field in 1992. The calculation is based on the information contained in *Oil Production Capacity in the Gulf, vol II: Saudi Arabia* (Center for Global Energy Studies, 1993). Annual infrastructure maintenance cost is about \$157 per barrel, or \$0.44 per daily barrel and thus total annual infrastructure maintenance as a function of total production is

$$\mu_t = 365 \cdot 0.44 X_t .$$

For adding one daily barrel of capacity, a \$1459.3 annual surface installation cost is required. In accordance with the model specification in (2.1), the surface installation cost for expanding capacity is represented in the cost of drilling a new producing well. For an Arabian light crude field this is about \$2 million per well. Hence the total cost of new wells is

$$\eta = 2dN_t .$$

Information on the maintenance cost for wells, as with the cost of the surface infrastructure, is hard to obtain. In the Black Oil simulation, a 20-year depreciation life is assumed for a producing well, which is in line with industry averages. In our numerical simulations, we assume that an oil well could not produce oil normally if its depreciated value is too low compared with its original value. Thus, to maintain a well's production a certain amount of investment is needed to offset the depreciation of equipment in the well. To keep the calculation simple, we assume that the well maintenance cost is evenly distributed over the life of a well and the depreciation rate is 10%. This gives rise to an approximate annual maintenance cost (in \$million) of

$$M_t = 0.1783 N_t$$

Production cost refers to operation costs and reservoir engineering costs. Operation costs denote the expenditures on manpower and other variable production costs. According to a report from the U. S. Energy Information Agency (EIA) (1996), variable operating expenses per barrel range from \$0.25 to \$1.00, depending on the rate of extraction. The EIA provides the following estimated functional relationship between production and variable operating costs[6]:

$$V_t = 0.7714(365X_t)^{-0.2423} \tag{3.3}$$

Water must be injected into the reservoir in order to maintain the reservoir fluid pressure. We use the water injection costs to capture the reservoir engineering costs. Industry studies indicate that these costs are on the order of about \$0.20/barrel of water per day for Saudi Arabia. This leads to an annual water injection cost of

$$\omega_t = 365 \cdot 0.20W_t$$

Summing these components, the estimated oil production cost (in \$million) per year for a typical oil field in Saudi Arabia is

$$C_t = 1.2\{\mu + V_t(365X) + \omega_t + M_t + \eta_t\} = \tag{3.4}$$

$$1.2\{\mu_t + V(0.7714(365X_t)^{-0.2423}) \times (365X_t) + 365 \times (0.2W_t) + 0.1783N_t + 2dN_t\}$$

where X, W, N, dN follow the same notation as in (2.1). The estimated water injection equation is modeled as a function (W) of the number of producing wells (N) and of the production rate (X). We provide a detailed discussion of how the production levels are modeled in the next section.

3.3 The Dynamic Production Function

Salehi-Isfahani (1995) provides a review of models of the oil market that updates earlier his earlier joint work in Crϑmer and Salehi-Isfahani (1991). In his review Salehi-Isfahani notes that "…Depending on the type of geological structure, oil may be lost due to pressure and seepage. Unfortunately, the economic literature has so far not incorporated much technical knowledge about the operation of the fields. Mining engineers often predict a production path from a given field as an inverted U, with a unique peak. Economists on the other hand emphasize the role of prices in extraction. Adelman (1993b) correctly criticizes the exhaustible resource models for their lack of realism in the description of reserves…". Below we attempt to address these concerns by formally modeling the dynamic engineering model of oil field fluid dynamics.

[6] The EIA's *Estimator* database contains field and production characteristics for eight geological plays (defined as a group of discovered and /or undiscovered fields with similar geological, geographic, and temporal characteristics) as well as varying field sizes based on expected ultimate recovery.

We use the *Workbench* Black Oil Simulator to model the dynamic production characteristics of an oil field with properties typical of a large Saudi reservoir. The predominate producing reservoir in the giant Ghawar field, like the majority of reservoirs exploited in producing Arab Light oil, is derived from the Jurassic Arab-D formation. Geological characteristics as well as detail on the black oil simulation exercise can be found in Appendix I.

The laws of physics control how fluids are distributed in oil reservoirs that are undisturbed for geologic time and how those fluids move in a reservoir once they are disturbed by the initiation of production and/or injection. Those laws of physics can be expressed as partial differential equations. The *Workbench* Black Oil Simulator starts with the fluids in place at the time of discovery, consistent with an equilibrium attained over geologic time. This becomes the set of initial conditions for the partial differential equations. Mathematical representations of wells then are added to the description and the partial differential equations are stepped through time to predict the movement of the fluids through the reservoir.

Wells can be used to produce fluid from the reservoir or to inject fluid into the reservoir. Technology can control only what happens at the wells and even then not absolutely. Nature controls everything in the reservoir between the wells. While technology can control what fluid is injected into a well, a producing well can produce only those fluids in its immediate vicinity in the reservoir. The Black Oil Simulator predicts how the fluids move in the reservoir based on the reservoir properties and how hard the wells are produced.

We accumulated the temporal production, water injection, and well drilling schedules from the simulations of the Workbench Black Oil software discussed in Appendix I. Using the simulation results, we then estimated two important functions that comprise our dynamic production function (at the full capacity) for the oil field – a water injection function and a short-term capacity function.

The water injection rate is modeled as a function of oil output and the number of producing wells as follows:

$$\log W_t = \delta_0 + \delta_1 \log X_t + \delta_2 \log\left(N_t^*\right) \qquad (3.5)$$

where $N_t^* = N_t + dN_t$, which is the total number of oil producing wells during period *t*.

OLS regression results are reported in Table 3.2.

Table 3.2: Estimation of the Water Injection Equation

	δ_0	δ_1	δ_2
Estimates	*0.7999*	*0.9509*	*0.0306*
Standard Deviation	*0.0284*	*0.0022*	*0.0007*
R^2	*0.9225*		

Given the results, we can rewrite the water injection function as,

$$W_t = e^{0.7999} X_t^{0.9509} N_t^{*0.0306} \tag{3.6}$$

The Black Oil Simulation program essentially applies the results from field dynamics to facts about the geological structures involved. The estimated relation (3.6) could thus be viewed as revealing the physical relationship between oil extraction and water injection in the field.

The dynamic[7] production function (at full capacity) of an average oil well in the field is modeled in the following semi-log form.

$$\max(X_t) = \lambda_0 + \lambda_1 \log W_t + \lambda_2 \log W_t \log CP_t + \lambda_3 \log CP_t + \varepsilon_t \tag{3.7}$$

where max(.) denotes the maximum producing capacity in the neighborhood of an oil well in the reservoir for a period. The estimation of (3.7) yields an approximated feasible set for oil production dependent on the reservoir engineering conditions.

Regression results are reported in Table 3.3.

Table 3.3: Estimation of the Dynamic Production Equation

	λ_0	λ_1	λ_2	λ_3
Estimates	*0.0451*	*0.0362*	*-0.0038*	*-0.0044*
Standard Deviation	*0.00023*	*0.00053*	*0.000064*	*0.000027*
R^2	*0.7950*			

The estimated coefficients are consistent with the hypotheses that short-term overproduction will jeopardize the producing environment of a particular well and also that water injection generally has a linear and positive relation to the amount of oil that can be extracted. If the cumulative production of a well is too high, however, it is possible that water injection could further reduce the short-term capacity as described previously.

Since the dynamic production function depends on the state variables in our optimization model and describes the feasible production set, equation (3.7), extended to the entire field, is used as a short-term capacity constraint in our model, which is function *f* in (2.1).

$$X_t \le f\left(W_t, N_t^*, CP_t\right)$$
$$= \left\{0.0451 + 0.0362\log\left(W_t\right) - 0.0038\log\left(W_t\right)\log\left(CP_t\right) - 0.0044\log\left(CP_t\right)\right\} \cdot N_t^* \qquad (3.8)$$

Note that the water injection equation (3.5) implies that the short-term field production capacity is not simply the function *f*. However, equation (3.7) implicitly determines the maximum amount of oil that can be produced from a typical Saudi oil field as a function of those reservoir state variables. We call (3.7) the 'short-term' field capacity function in part because the next period state variables (*CP* and *N*) are functions of decision makers' choices, and thus evolve over time. In addition, W_t is a time-varying indicator for the reservoir fluid pressure and is affected by the choice variables directly and indirectly.

3.4 Summary

The results derived previously in this section can be summarized by restating the estimated dynamic programming model as follows. Notice that the first two equations in the constraints represent the state transition equations in the model.

$$V_t\left(N_t, CP_t\right) = Max_{X_t, dN_t}\ \{\left(365X_t\right)\cdot e^{3.5323-0.0398\times 37\times X_t+3.9656t} - 1.2\{(0.44\times 365)X_t + \qquad (3.9)$$
$$(0.7714(365X_t)^{-0.2423})\times(365X_t) + 365\times(0.2W_t) + 0.1783N_t + 2dN_t\} + \beta V_{t+1}\left(N_{t+1}, CP_{t+1}\right)\}$$

subject to

$$N_{t+1} = N_t + dN_t$$
$$CP_{t+1} = CP_t + 365X_t$$
$$W_t = e^{0.7999} X_t^{0.9509} N_t^{*0.0306}$$
$$X_t \le \left\{0.0451 + 0.0362\log\left(W_t\right) - 0.0038\log\left(W_t\right)\log\left(CP_t\right) - 0.0044\log\left(CP_t\right)\right\} \cdot N_t^*$$

[7] 'Dynamic' here refers to the effects of current production on future capacity and choices of policy variables.

$X_t \geq 0$ and $dN_t \geq 0$

The final piece of information needed to derive the optimal policy path is the functional form of the value function V. In the literature, different iterative methods have been used to approximate the value function. For example, Hartley (1996) proposes polynomial approximation of the value function in models with inequality constraints as we have here. Our case is a little different from his though, in so far as we have multiple state variables. Therefore, we use a nonparametric technique, tensor splines, to approximate the value function on two-dimensional grids and modify his approach to handle the inequality constraint. These issues are discussed further in Section 5.

4. Existence and Uniqueness of the Optimal Stationary State Solution

We shall numerically approximate the solutions to the dynamic programming model in (3.9). First, we need to address the existence and uniqueness of the optimal solution in the time-invariant state to ensure that the iterative process used to approximate the value function is valid.

The value function in (3.9) depends on time, t, because the revenue function is time varying. In order to apply iterative techniques to solve for a value function we need to hypothesize that the choice environment eventually settles down to a stationary one that depends only on the state variables and the parameters of the model. The state variables and optimal policies will not be constant in this regime, but they will be stationary functions of the time-varying state variables. In two of the four simulation scenarios, a competitive backstop technology (solar energy) is assumed to control the demand for fossil fuel in the time-invariant terminal control problem. In the other two scenarios, demand is simply assumed to become stationary at some date. In each case, the iterative procedure yields a "terminal value function" for oil revenues as a function of the state prevailing at the time we enter that stationary world.

In the remainder of this section, we focus on the feasibility of the iterative procedure that we use to solve for the optimal policy and value functions in the terminal time-invariant control problem. We will use the solution to the terminal optimization problem to solve the problem for optimal policy and value functions in the earlier periods using a finite number of backward recursions of (3.9).

First we introduce some notation. Let π denote the policy variables and σ the state variables. Π is used to represent the set of all possible values for the policy variables (i.e., $\pi \in \Pi$), and Σ stands for the set of all possible values for the state variables (i.e., $\sigma \in \Sigma$). Let Φ: $\Sigma \rightarrow \Pi$ be the correspondence describing the feasibility constraint. Notice that the correspondence Φ here is implicitly defined by the short-term capacity function (3.8) as an upper bound and the nonnegativity of the policy variables as a lower bound. $\hat{\mathrm{B}}$ denotes the estimate of the profit function ("$\wedge$" stands for the estimated function) that is equal to the estimate of $r - c$ in (2.1). Hence $\hat{\mathrm{B}}$ is a function mapping from $\Pi \times \Sigma$ to $\mathbb{R}$. M stands for the state transition function, M: $\Pi \times \Sigma \rightarrow \Sigma$, which is described in (3.9).

Given the notations defined as above, the dynamic programming model in (3.9), once the choice environment is stationary, can be summarized as (4.1) or (4.2) in a relatively abstract way[8], with an operator, *T*, on the space of continuous and bounded functions.

$$(T \cdot v)(\sigma_t) = \max_{\pi_t \in \hat{\Phi}(\sigma_t)} \left\{ \hat{\mathrm{B}}(\pi_t, \sigma_t) + \beta v(\sigma_{t+1}) \right\} \tag{4.1}$$

or

$$(T \cdot v)(\sigma_t) = \max_{(\sigma_{t+1} -_* \sigma_t) \in \hat{\Phi}(\sigma_t)} \left\{ \hat{\mathrm{B}}(\sigma_{t+1} -_* \sigma_t, \sigma_t) + \beta v(\sigma_{t+1}) \right\} \tag{4.2}$$

where $-_*$ indicates that the subtraction follows the state transition function M.

Theorem 4.1 *Let $\mathbb{C}(\sigma)$ be the space of bounded and continuous functions g: $\Sigma \rightarrow \mathbb{R}$, with sup norm. Then operator T defined in (4.1) has a unique fixed point $v^* \in \mathbb{C}(\sigma)$, and for all $v_0 \in \mathbb{C}(\sigma)$, such that*

$$\left\| T^n v_0 - v^* \right\| \le \beta^n \left\| v_0 - v^* \right\|, \qquad n = 0, 1, 2, \ldots.$$

Moreover, given v, the optimal policy correspondence P: $\Sigma \rightarrow \Pi$, defined as*

$$\mathrm{P}(\sigma) = \left\{ \pi \in \Phi(\sigma) : v^*(\sigma) = \hat{\mathrm{B}}(\pi, \sigma) + \beta v^*(M(\pi, \sigma)) \right\}$$

is compact-valued and upper hemi-continuous (u.h.c.).

(Proof : see Appendix II).

Theorem 4.1 shows the existence of a fixed point for the operator *T* and hence allows us to use iterative methods to approximate the value function in the time-invariant state for the model in (3.9). The theorem also provides a bound on the rate of convergence. The following corollary examines the monotonicity of the value function *v** in the time-invariant state. We first need to

[8] The equivalence of (4.1) and (4.2) is demonstrated in the proof for Theorem 4.1 in the Appendix.

rewrite one of the state variables so that standard methodology can be employed. We denote CP^{-} as $CP^{-} = - CP$ and substitute $CP = - CP^{-}$ back into the dynamic programming model (4.1) as part of σ. All the functions involving CP are redefined as a function of CP^{-}. This transformation obviously has no effect on the existence of v^* and the value of the optimal policy.

Corollary 4.1 *Let v^* be the unique solution to our dynamic programming model (4.1) in the time-invariant state. Then v^* is strictly increasing in both state variables N and CP^{-}.*
(Proof: see Appendix II)

Hence, v^* should be strictly increasing in N and strictly decreasing in CP. The latter follows from the transformation of CP to CP^{-}. The simulated value functions, which are presented in the next section, are consistent with Corollary 4.1.

5. Simulation Results

This section presents numerical approximations to the value function and the optimal policy paths in four simulated time-invariant environments. As we noted above, these environments represent different hypotheses about the transition from fossil fuels to backstop energy technologies. Scenario I assumes a discount factor equal to 0.9 (discount rate $\simeq$ 0.1) and the time-invariant states (backstop technology) beginning with the level of demand attained in 2045. Scenario II keeps the backstop replacement date as 2045 but sets the discount factor to 0.7 to capture the heavy reliance of Saudi Arabia on its short-term oil income. In effect, oil reserves are risky as an asset and hence the implicit rent from leaving oil in the ground has to yield a substantial risk premium. The optimal extraction policies under the two discount rates thus reflect possible effects on long-term oil extraction policy of relying on an undiversified income structure. Scenario III studies the impact on Saudi Arabia's oil policy of a technology breakthrough that significantly reduces the cost of solar energy. The alternative technology is assumed to become competitive with oil in 2036 and continue to improve and reduce oil demand. We allow a ten-year period for solar energy to totally displace the demand for Saudi Arabia crude oil in terms of energy consumption. In the meantime, the demand for oil from the petrochemical industry is assumed to remain unchanged from current levels, which are about 5% of the total demand for Saudi Arabian oil. Only the revenue function is affected by such a shock to oil demand. The transition pattern is purely hypothetical. We assume that the demand for Saudi oil decreases by 5% at the year of the breakthrough, and then declines 10% every year for another nine years. The exact pattern of the demand displacement is not the focus of our analysis. Rather, we are interested in how expectations of such a transition would affect Saudi Arabia's oil production policy in the short to medium term. Scenario IV is a repetition of scenario III with a different timing of the solar technology shock. We assume the more optimistic Department of Energy projection discussed above that implies the breakthrough will occur in 2026 instead of 2036. This allows us to see how the expected timing of such a shock affects Saudi's oil production decisions. Note that changing the timing of the solar technology shock from 2036 to 2026 should have no effect on the value function v^* in the time-invariant state, though the values of the state variables at the time of the shock of course would be different.

Figures 5.1-5.3 graph the tensor spline numerical approximations to the value function in the terminal time-invariant state for each scenario.[9]

[Figures 5.1-5.3 here]

The value functions in earlier periods are derived using backward recursion. This gives us a time-varying value function at each earlier date. We then solve the optimization problem forward from the starting states using the previously calculated value functions at each date.

Figures 5.4-5.11 present the production paths, and corresponding trajectories of new wells drilled in the field, that yield the optimal oil production policy for each scenario. The initial conditions for the simulated oil field are assumed as: $N_0 = 57$ and $CP_0 = 2648$ million barrels. The numbers are set to be about 10% of their actual values in Saudi Arabia, which is also due to the fact that the original Black-Oil simulation was conducted for an oil field about this size.[10] CP_0 is calculated using the Saudi's production data 1976–1985[11].

There are several interesting conclusions one can draw from the results. First, the discount rate has a significant impact on the optimal extraction rates and the number of new wells drilled for

[9] Due to their flexibility and smoothness, splines are widely applied in approximating functions and solving functional equations. The value function approximations and subsequent simulations for the optimal policies are conducted using the Spline toolbox and Optimization toolbox in MATLAB.

[10] The simulated oil field only accounts for part of the Ghawar field in terms of its production and number of wells. However, it still represents Ghawar, in the sense that the similar geological features and field properties assumed in the Black Oil simulations match those of the Ghawar field.

[11] See *PennWell Publishing* (1997). Since we have no exact data on the total number of on-shore producing oil wells in 1986, we approximated it using the information that Saudi has about 555 producing wells during the mid-1980's. The information was obtained from the web site: http://lcweb2.loc.gov/cgibin/query/Dcstdy;2:/temp/~frd_FXn3. The original data source is The U.S. Library of Congress Country Studies.

capacity expansion. Comparing scenario I where $\beta = 0.9$ with scenario II where $\beta = 0.7$, we find a higher production rate in the short run, but lower capacity in the long run, in the case with a lower discount rate. With a higher discount rate, the present value of future oil income is relatively lower while the present value of the cost of drilling a well is higher. The second key finding is that the expectation of a future breakthrough in solar energy technology tends to increase production for a short period and reduce capacity subsequently. It operates somewhat like the effect of a smaller discount factor. This is indicated by the similarity between a lower β of scenario II and the backstop energy scenarios III and IV. In both cases, the opportunity cost of current production in terms of future oil income is reduced compared with scenario I. The different assumptions in scenarios II and III have different effects, however, on the number of new oil wells drilled for capacity maintenance and expansion. The decision makers in scenario II would be much more concerned about short-term oil profits and much less concerned about future production than decision makers in scenarios III and IV, who favor a relatively higher production capacity and thus more extraction. Under scenarios III and IV, decision-makers foresee a sharp decline in the return on oil reserves due to the substitution of an alternative energy for oil. Finally, under scenario IV, where Saudi Arabia's decision makers expect the technology shock to occur earlier, short-run production rates would be even higher (compared with previous scenarios) while production capacity would be maintained and even expanded (for a period) by drilling more oil producing wells.

In Figure 5.12 the simulated optimal production paths (1986–1995) in different scenarios are compared with 10% of the Saudi actual daily extraction rates during this period. The 10% level is based on the size of the oil field studied in the Black-Oil simulation and the fact that the initial conditions were chosen under the assumption that the field accounts for about 10% of total Saudi oil reserves and production**.** Figure 5.12 suggests that Saudi oil production is better approximated by scenario I ($\beta = 0.9$) before 1990. Since 1990, the year of the last Gulf war, scenario II ($\beta = 0.7$) approximates the real path relatively better. This is not difficult to explain if we recognize that there was a large increase in Saudi government expenditure (especially military expenditure) around 1990, as pointed out by Azzam (1993). In fact the path in scenario II in 1990 coincides with the actual one.

[Figure 5.12 about here]

Another interesting question is what happens to the oil price paths in each of the four scenarios and how do they compare to the actual Saudi price path? This is illustrated in Figure 5.13.

[Figure 5.13 about here]

We find that the actual price path is more volatile than the four simulated price paths. While the simulated price paths should be thought of as the mean of the potential sample paths, the volatility of the actual path suggests that other factors affecting Saudi Arabia's decision making process are omitted from our model. The contrast between the apparent flat trend in the actual price path and the rising trend in the simulated paths is somewhat puzzling[12]. Quite possibly it reflects a slower growth in the world economy, and hence in energy demand, than the OMS model had anticipated. Another apparent feature from the price comparison is that the simulated oil prices approximate the actual ones much better before 1990 than after 1990. After 1990 all the price simulations lie above the real price path. This is not surprising either given that the true oil supply from Saudi Arabia is higher than the simulated optimal paths (after 1990) in all scenarios (see Figure 5.12).

6. Conclusions

We have proposed and illustrated an economic and engineering-based methodology to model the dynamic production decisions from an idealized oil field. The field we chose approximates the largest oil field in Saudi Arabia – Ghawar. Our analysis incorporates both a game theoretical structure of the world oil market with the OMS data and the reservoir engineering operations through the Black Oil simulation. The results of the optimal production model approximate actual Saudi Arabia extraction rates. Perhaps of more interest, a comparison of the results under different simulation scenarios helps elucidate the possible effects of Saudi Arabia's dependence on risky oil income, and the effects of possible future demand reductions resulting from the development of alternative energy technologies.

---

[12] The rising trend of the simulated price path per se, however, is not surprising at all given the estimated demand equation. It is also consistent with the prediction of Hotelling's condition. For details see Prato (1997, p. 140) or Neher (1990, p. 95).

It is well known that Saudi Arabia's oil policy reflects non-economic objectives. The simulation results allow us to evaluate the potential costs of pursuing those objectives in terms of foregone profits. We present the magnitude of potential profit losses in the following table. Table 6.1 presents revenue losses as a percentage of revenues under Scenario I, the lowest one in all simulation scenarios during the comparison period.

We observe a substantial potential for revenue (and hence profit) losses from pursuing what appears to be less than optimal policy from a purely profit maximizing perspective. However, there are a couple of years (1990–1992) in which the actual revenues are higher than any of the simulated ones during the same period. The possible explanation is that the exogenous shock of the Gulf War raised oil prices (for a short period) as well as Saudi Arabia's oil sales, as can be seen in Figure 5.12 and Figure 5.13. Our model does not take into account the impacts of such shocks.

Table 6.1: A Comparison of Simulated Oil Revenues (million dollars) with the Observed Levels

	Scenario I	Scenario II	Scenario III	Scenario IV	Real Rev.	Losses
1986	32004	33171	33016	33315	28489	-11%
1987	34612	35456	34899	35498	25739	-26%
1988	37207	37891	37877	37788	23720	-36%
1989	39924	40493	40473	40618	33526	-16%
1990	42744	43268	43112	43397	55188	29%
1991	45710	46236	46116	45978	47345	4%
1992	48844	49406	49327	47664	51574	6%
1993	52119	52784	52707	52588	35787	-31%
1994	55665	56390	56297	56188	46985	-16%
1995	59545	60159	60137	60044	52242	-12%

Although the model appears to provide a reasonable approximation to Saudi Arabia's oil production decisions in its largest light oil field, we have made many simplifications that could be relaxed in future work. First, as pointed out by Powell (1990), the assumption of perfect knowledge and foresight is unrealistic. While we tried to account for unknowns and uncertainties by examining a number of different scenarios, we would like to extend our model in the future to encompass a stochastic demand environment and randomize the timing of the breakthrough in solar technology. Second, instead of modeling the Saudi Arabia oil production decision purely as maximization of the present value of its profits, we could apply a multi-level optimization (MLO) approach to incorporate profit maximization by local oil firms along with other motivations of the Saudi Arabian government. This type of approach might approximate the Saudi Arabia's oil production decision more accurately. Islam (1998) provides a good example of using MLO to model energy plans involving both the private sector and the government. Last, but not the least, while the OMS model allows us to simplify our modeling task, the simplification might be costly in terms of the accuracy of the model. A more ambitious model might attempt to improve upon the OMS model of the world oil market and incorporate strategic (market) behavior directly into the dynamic optimization model.

Appendix I – Geological Characteristics and the *Workbench* Black Oil Simulation Strategy

The Jurassic Arab-D formation is a calcarenitic limestone more akin to a highly porous and permeable sandstone than a fractured limestone. This formation is the predominant producing reservoir in the giant Ghawar field.

The simulator model is set roughly 5 kilometers from the crest to the oil/water contact. The dip of the formation varies from 0 degree at the crest to about 5 degree on the flank. The wells are drilled in a square pattern with 1 kilometer between nearest neighbors. The reservoir gross thickness is about 250 feet. The simulation model is a 3-dimensional wedge perpendicular to a line along the crest with its sides passing through adjacent lines of wells (0.707 kilometers apart). The model extends downdip to below the oil/water contact. Water injection wells are located below the oil/water contact for peripheral water injection. The rock properties, the fluid properties, and the fluid/rock interaction properties were set to values that are typical for reservoirs in the neighborhood of the Ghawar field. Figure A1 shows a cross section perpendicular to a line along the crest and through a line of wells. The vertical scale has been exaggerated relative to the horizontal scale in order to display details in the layering. This figure shows the initial positions of the fluids in the well.

For example, Figure A2 shows what would happen if the reservoir were produced with no water injection. The direct result is the loss of reservoir pressure. Therefore, oil production is reduced considerably, because gas is produced preferentially over oil or water. Figure A3 shows what would happen if water injection and fluid production balance. In this case, much more oil is produced because water effectively displaces and takes the place of oil.

[Figures A1-A3 about here]

The results of the Black Oil Simulator consist of time series of the pressure and the amount of oil, gas, and water present at selected points in the reservoir and the production rates of oil, gas, and water out of, and injection rates of water into, each of the wells. Each of these properties can be presented as tables of numbers or in graphical displays. The graphical displays aid understanding of the physical processes occurring in the reservoir. The mountain of tabular data can be used for statistical calculations. The pressure and the amount of oil, gas, and water present in the reservoir describe the “state” of the reservoir. Correspondingly, the pressure and the amount of oil, gas, and water present in the reservoir around a well and the pressure in the well determine the ability of the well to produce or inject fluids. Thus, the performance of the well can be taken as a measure of the "state" of the reservoir. However, the performance of the well does not include the "state" of the entire reservoir, but just the area around the well. For this reason, correlating the performance of the producing wells against that of the injection wells is not always successful.

The typical operational procedure in Saudi Arabia is to drill producing wells downdip of the crest and injection wells below the oil/water contact. Only enough producing wells are drilled so as to meet the targeted production rate. The injected water travels across the reservoir until it encroaches upon the producing wells. When water reaches the producing wells, the oil production rate falls. Eventually, the wells are not able to meet the production target and new wells have to be drilled updip to maintain capacity. Successive wells are drilled until the wells at the crest of the formation water out. A suite of simulations was run to investigate the performance of the reservoir for a range of production and injection levels. The amount of production and injection was balanced in each run. Any changes to the rates were made to the producers and injectors at the same time.

The scheduling of the wells in the model takes place in the following fashion. There are seven wells in the model: five producing wells and two injection wells. The sequencing of the wells begins with the producer lowest on the structure and ends with the fifth producer, which is drilled on the crest. The injectors are both drilled below the oil/water contact. Both of the injectors, as well as the first producing well, are drilled at the same time and thus represent the initial investment. Subsequent capital investments are made as the additional producers are drilled in response to the watering out of previous wells. These wedges of the reservoir then are horizontally stitched together side by side to approximately cover the Ghawar reservoir's northern portion of Aindar. The eighty-four wedges in the model cover an area of about 10×40 kilometers, which is comparable to Aindar's 10×30 kilometer size. The main portion of Ghawar is about 180 kilometers wide. Reservoirs that are close by, and which are comparable to the representative reservoir we model, include Abuaiq (15×40 kilometers) and Harmaliyah (8×15 kilometers). The production and injection rates were held constant throughout each simulation. The length of the simulation is 23 years. At the lowest level of production target where oil production rates are 5% of total reserves per year (Figure A4), only one well is needed to meet the target rate. The nominal dates in the figure are a convention of the workbench software. As the well waters out, one new well is drilled. However, the deliverability of the next to highest well is insufficient to meet the target and the

highest well has to be drilled at the same time. At the highest level of production target (9% of reserves per year), only the first well is able to meet the target rate by itself. When the first well waters out, all the remaining wells have to be drilled. At the intermediate production rate targets (6%, 7%, 8% of reserves per year), the results lie in between these two extremes. In general,
the higher the target production rate, the quicker the wells have to be drilled. The capital investment schedule for each of these cases would be different, because of the different schedules for drilling the wells. There is a trade-off between deferring capital investment at lower production rates against the consequent loss of revenue.

Twenty-four additional sets of simulations were performed to examine the effects of production rates on well requirements and water injection. Each simulation was started at one of the five levels of production target just described. After a length of time at that initial rate, the rate was either increased to the next higher, or decreased to the next lower, level of production target and then maintained at that rate for the remainder of the conducted. Simulations were made with the rate change at two, four, and six years. In all, twenty-nine simulations were run.

[Figure A4 about here]

Appendix II – Proofs

Proof for Theorem 4.1

We first need to show that our estimate of the dynamic programming model (4.1) satisfies the following two assumptions.

(A1) Σ *is a convex subset of* $\mathbb{R}^2$, *the correspondence* $\Phi: \Sigma \rightarrow \Pi$ *is nonempty, compacted-valued, and continuous.*

(A2) *The function* $\hat{\mathrm{B}}: \Pi \times \Sigma \rightarrow \mathbb{R}$ *is bounded and continuous, and* $0 < \beta < 1$.

The first part of (A1) is not restrictive for our simulations[13]. *The second part requires the maximum number of new oil wells that can be drilled within one period to be bounded, which is reasonable because in reality the number of new wells that can be drilled during a period has to be bounded by economic and engineering constraints*[14]. *Then given that* $\hat{f}(\pi,\sigma)$, *defined by (3.8), is a bounded and continuously differentiable function, it is not difficult to verify that the short-term capacity function is nonempty, compact valued, and continuously differentiable by the Implicit Function Theorem, and hence so is the correspondence* $\Phi: \Sigma \rightarrow \Pi$, *given* Σ *is compact. The continuity of the function* $\hat{\mathrm{B}}(\pi,\sigma)$ *follows directly from the linear regression results used in the estimated dynamic programming model (3.9). With (A1) and the assumption that* Σ *is compact (bounded and closed),* $\hat{\mathrm{B}}(\pi,\sigma)$ *is also bounded. Therefore, (A2) holds in general.*

Next we demonstrate the equivalence between (4.1) and (4.2) by showing that, in (4.1), the state variables in the following period can also be viewed as the choice variables. In particular, there exists a correspondence $\dot{\Phi}: \Sigma \rightarrow \Sigma$ *that also satisfies assumption (A1). For any elements* $\sigma_0 \in \Sigma$, *there is a set in* Π *defined by correspondence* Φ *and denoted as* Λ, *which is nonempty and compact due to assumption (A1). We can further define a set in* Σ *denoted as* Δ *(i.e.,* $\Delta \in \Sigma$*) such that*

Δ = {$\sigma \in \Sigma$: σ *can be expressed as a summation of* σ_0 *and any element of* Λ, *and the summation is conducted according to the rule defined by function M*}.

[13] It is assumed in the numerical optimization that the number of wells can be any nonnegative real number. However, after the optimal solution is obtained in each step of the simulation, the number of new wells is rounded up in approximating the optimal new wells drilled.

[14] In all the simulations, we assumed that an upper bound for *dN* is equal to forty and the simulation results did not get even close to that number. Empirically, the feasible set for the number of new oil wells is bounded and closed.

Given σ_0 *and* Δ*, choosing* π *(*$\pi \in \Pi$*) is equivalent to choosing an element in* Δ *as the state variable of the next period and hence a correspondence* $\dot{\Phi} : \Sigma \rightarrow \Sigma$ *can be set up such that it assigns to each element of* Σ *an image set, similar to* Λ *defined for* σ_0*. Correspondence* $\dot{\Phi}$ *satisfies (A1) simply due to the way it is constructed.*

Then by Theorem 4.6 in Stokey, Lucas, and Prescott (1989), Theorem 4.1 holds for model (4.2), and hence (4.1). The basic idea of the theorem in Stokey, Lucas, and Prescott is to show that, under the assumption (A1) and (A2), T satisfies the hypothesis of Blackwell's sufficient conditions for a contraction mapping.

■

Proof for Corollary 4.1

The proof requires us to verify assumptions (A1) and (A2) and the following two new assumptions.

(A3) $\hat{B}(\cdot\,,\cdot)$ *in (4.2) (as defined in (3.9)) is strictly increasing in each of its state variables of the current period (i.e.,* σ_t*).*

(A4) Φ *is monotone in each of the state variables in the sense that* $\sigma \leq \sigma'$ *implies* $\Phi(\sigma) \subseteq \Phi(\sigma')$*.*

(A4) is easy to check for (4.1) (or (3.9)). The idea is that an increase in the number of producing wells or a decrease in cumulative production raises short-term capacity. Hence the image set of $\Phi(\sigma)$ *enlarges. This is consistent with the estimated short-term capacity function in (3.8), as a function of N and* CP^{-}*. (A3) is relatively more complicated to check because it requires us to use the next period state variables (*σ_{t+1}*) as the choice variable as modeled in (4.2). However, if we substitute the state transition equations (3.9) into the corresponding Bellman equation to eliminate the choice variables (*X_t *,* dN_t*) as required by (4.2), it is not difficult to demonstrate that the partial derivatives of the profit function* $\hat{B}$ *with respect to* N_t *and* CP^{-}_t *are in general positive. Therefore, with (A1), (A2), (A3), and (A4), Corollary 4.1 follows directly from our Theorem 4.1 and Theorem 4.7 in Stokey, Lucas, and Prescott (1989).*

■

References

Adelman, M. A. (1993a), *The Economics of Petroleum Supply*, Cambridge: The MIT Press.

Adelman, M. A. (1993b), "Modelling World Oil Supply," *The Energy Journal*, 14, 1-31.

Askari, H. (1991), "Saudi Arabia's Oil Policy: Its Motivation and Impact," in: Wilfrid Kohl (ed) *After the Oil Price Collapse: OPEC, the United States, and the World Market*, Baltimore: The Johns Hopkins University Press.

Azzam, H. T. (1993), *Saudi Arabia: Economic Trends, Business Environment, and Investment Opportunity*, London: Euromoney Books.

Benkherouf, L. (1994), "A Generalized Oil Exploration Problem*," European Journal of Operational Research*, 73, 423-429.

Center for Global Energy Studies (1993), *Oil Production Capacity in the Gulf*, Volume 2.

Cody, G. D., and T. Tiedje (1992), "The Potential for Utility Scale Photovoltaic Technology in the Developed World, 1990-2010"**,** In: Abeles, B., A. J. Jacobson, and P. Sheng (eds.) Energy and the Environment, 147-217, New Jersey, Singapore: World Scientific.

Crϑmer, J., and D. Salehi-Isfahani (1989), "The Rise and Fall of Oil Prices: A Competitive View," *Annales D'Economie et de Statistique*, 15/16, 427-454.

Crϑmer, J., and D. Salehi-Isfahani (1991), *Models of the Oil Market*, London: Harwood Academic Publishers.

Dahl, C., and M. Yucel (1991), "Testing Alternative Hypotheses of Oil Market Behavior," *The Energy Journal*, 12, 117-39.

Dracker, R., and P. De Laquil III (1996), "Progress Commercializing Solar-Electric Power Systems," *Annual Review of Energy and the Environment,* 21, 371-402.

Energy Information Administration (1996), *Oil Production Capacity Expansion Costs for the Persian Gulf*, DOE/EIA-TR/0606, Washington, D. C.

Energy Information Administration (1996), *The 1992 Oil Market Simulation Model*, Washington, D. C.

Energy Information Administration (1996), *International Energy Outlook*, DOE/EIA-0484, Washington, D.C.

Fousekis, P., and S. E. Stefanou (1996), "Capacity Utilization Under Dynamic Profit Maximum", *Empirical Economics,* 21, 335-359.

Griffin, J. M. (1977), "The Econometrics of Joint Production: Another Approach*," Review of Economics and Statistics*, 109, 389-397

Griffin, J. M. (1978), "Joint Production Technology: the Case of Petrochemicals," *Econometrica*, 46, 379-396.

Griffin, J. M. (1985),"OPEC Behavior: A Test of Alternative Hypotheses," *American Economic Review*, 75, 954-63.

Griffin, J. M., and Nielson (1994), "The 1985-86 Price Collapse and Afterwards: What Does Game Theory Add?," *Economic Inquiry*, October, 543-61.

Hartley, P. R. (1996), "Value Function Approximation in the Presence of Uncertainty and Inequality Constraints: an Application to the Demand for Credit Cards," *Journal of Economic Dynamics and Control*, 20: 63-92.

Hotelling, H. (1931), "The Economics of Exhaustible Resources," *Journal of Political Economy*, 39, 137-175.

Islam, S. M. N. (1998), *Mathematical Economics of Multi-level Optimization: Theory and Application*, New York: Physica-Verlag.

Khadduri, B. (1996), "Oil and politics in the Middle East security dialogue," *Security Dialogue,* 27, 155-166.

Lohrenz, P. E., and A. J. Bailey (1995), "Evidence and results of present value maximization for oil and gas development projects*," Proceedings, Hydrocarbon Economics and Evaluation Symposium*, Dallas, 163-177.

Masseron, J. (1990), *Petroleum Economics*, Institut Français du Pétrole Publications (Distributed in the United States by Gulf Publishing Company).

Neher, P. A. (1990), *Natural Resource Economics: Conservation and Exploitation*, Cambridge: Cambridge University Press.

PennWell Publishing Company (1997), *Energy Statistics Sourcebook, 12th ed.*

Powell, S. (1990), "The Target Capacity-Utilization Model of OPEC and the Dynamics of the World Oil Market," *The Energy Journal*, 11, 27-61.

Prato, T. (1997), *Natural Resource and Environmental Economics*, Iowa State University Press, Ames.

Quandt, W. B. (1982), "Saudi Arabia's Oil Policy: A Staff Paper," Wahsington, D. C.: The Brookings Institute.

Salehi-Isfahani, D. and J. Crémer (1991), "Models of the Oil Market," in *Fundamentals of Pure and Applied Economics*, 44, London: Harwood Academic Publishers.

Salehi-Isfahani, D. (1995), "Models of the Oil Market Revisited," *The Journal of Energy Literature*, 1, 3-21.

Scientific Software Intercomp (1995), *Workbench-the Petroleum Workbench-Version 1.6*, Denver, Colorado.

Stokey, N. L., R. E. Lucas, Jr., and E. C. Prescott (1989), *Recursive Methods in Economic Dynamics*, Harvard University Press, Cambridge, MA.

Suranovic, S. M. (1993), "Does a Target-Capacity Utilization Rule Fulfill OPEC's Economic Objectives?," *Energy Economics,* 15, 71-79.

Teece, D. (1982), "OPEC Behavior: An Alternative View," in J. J. Griffin and D. Teece, eds., *OPEC Behavior and World Oil Prices*, London: Allen and Unwin.

Wirl, F. (1990), "Dynamic Demand and Optimal OPEC Pricing: an Empirical Investigation," *Energy Economics,* 12, 174-176.

Figures

Figure 5.1: Approximated Value Function in the Time-Invariant State

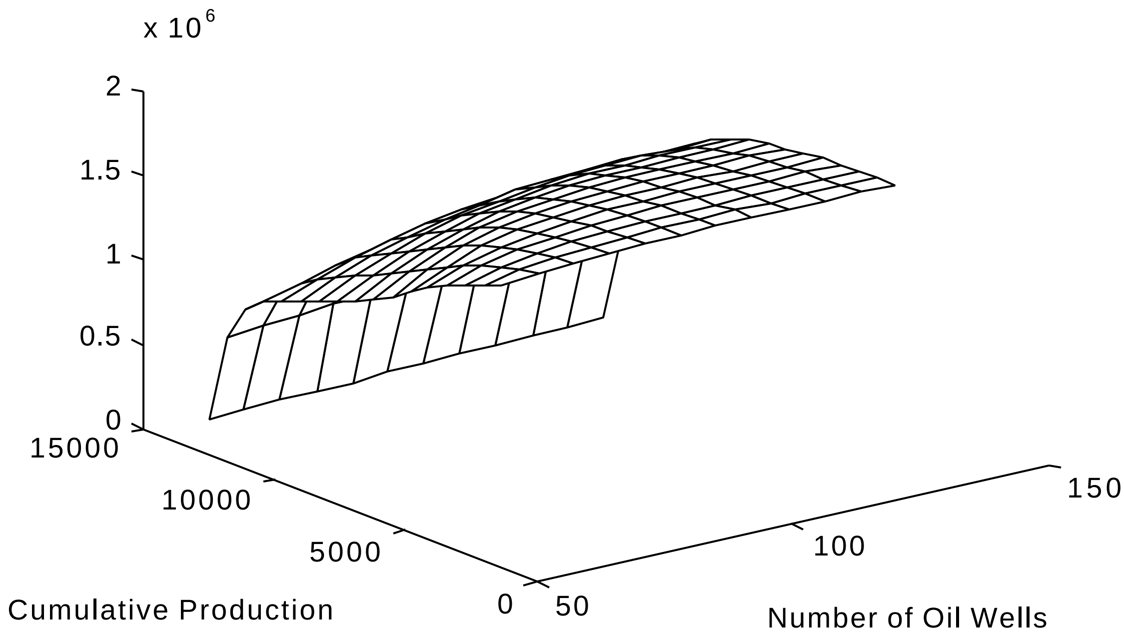


Figure 5.1 depicts the approximated value function in the time-invariant state under scenario I with no assumed solar technology shock and the discount factor b = **0.9**.

Figure 5.2: Approximated Value Function in the Time-Invariant State

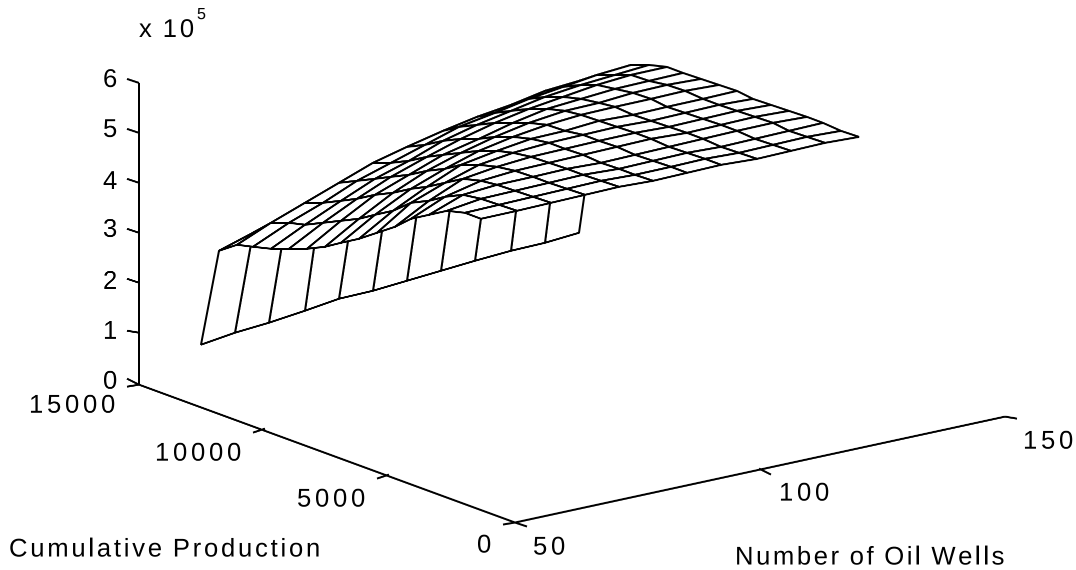


Figure 5.2 depicts the approximated value function in the time-invariant state under scenario II with no assumed solar technology shock and the discount factor *b* = **0.7**.

Figure 5.3: Approximated Value Function in the Time-Invariant State

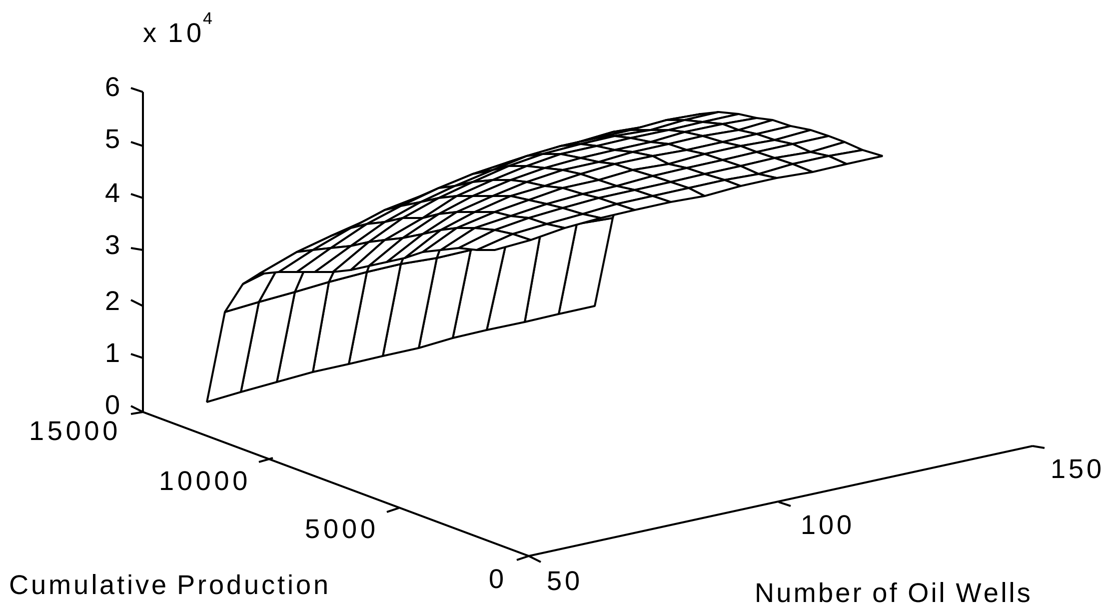


Figure 5.3 depicts the approximated value function in the time-invariant state under scenarios III and IV.

Figure 5.4: Optimal Extraction Rates in Scenario I

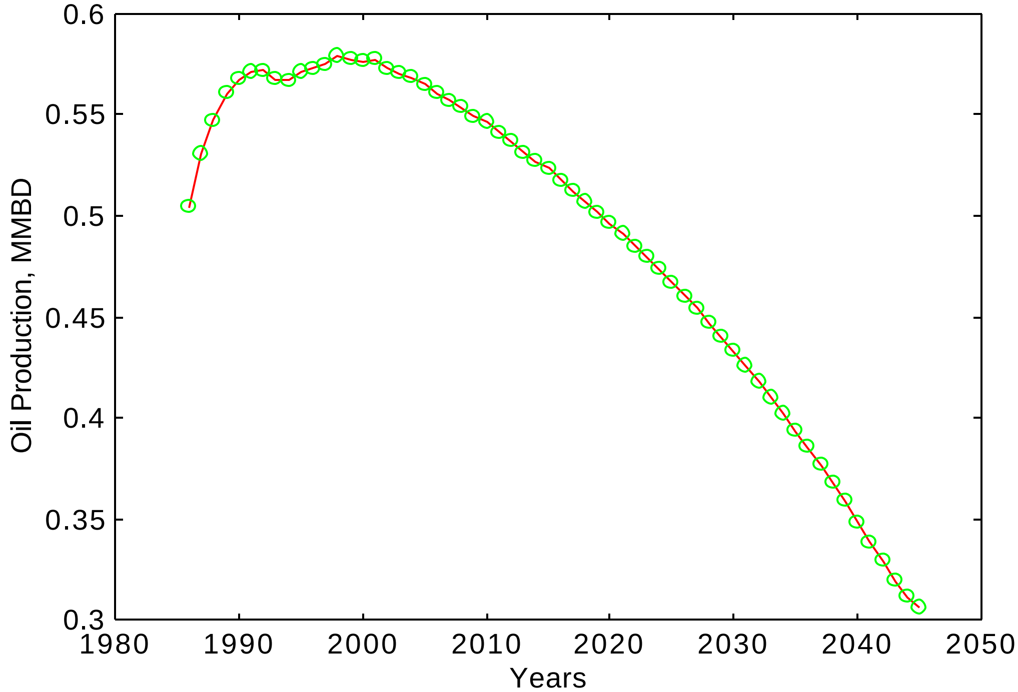
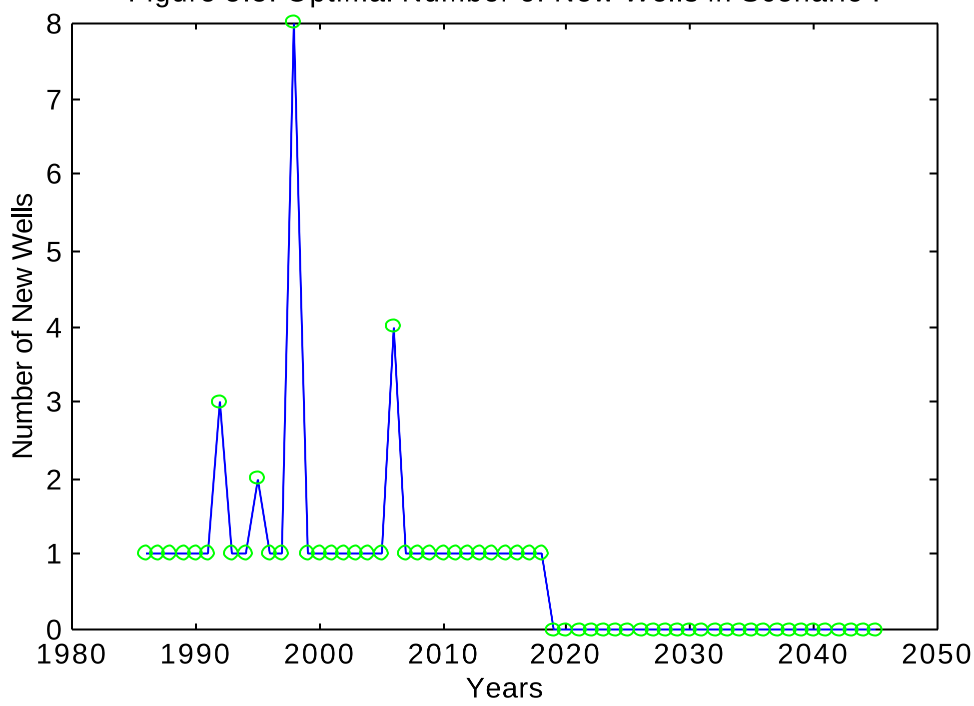


Figure 5.5: Optimal Number of New Wells in Scenario I



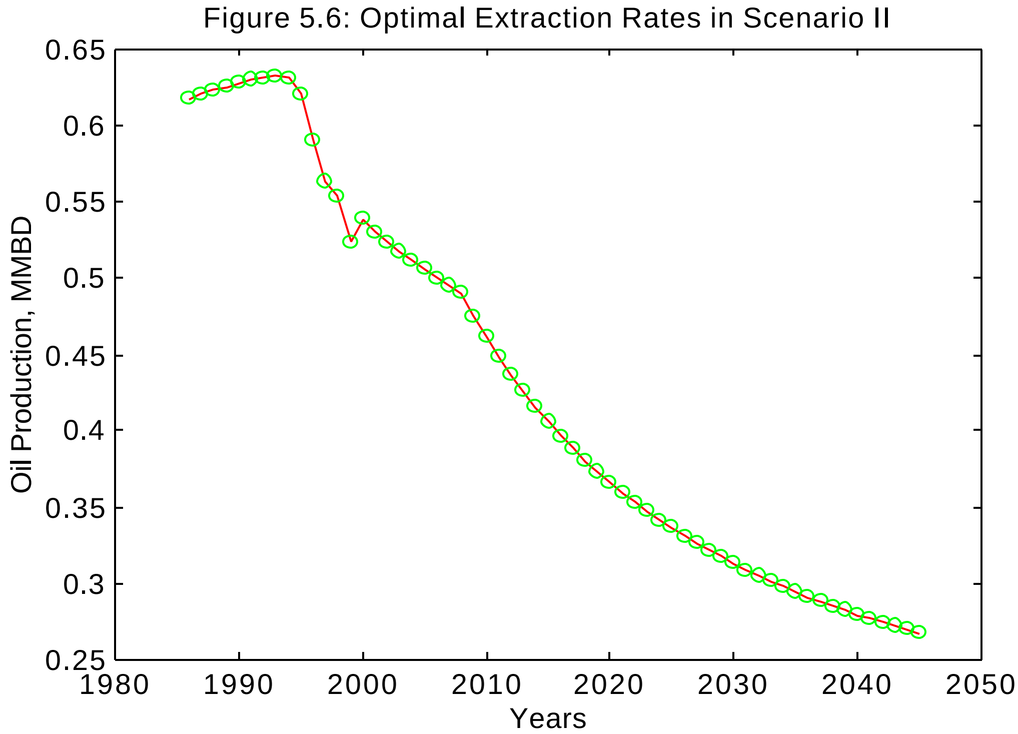


Figure 5.6: Optimal Extraction Rates in Scenario II
Oil Production, MMBD
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
1980
1990
2000
2010
2020
2030
2040
2050
Years

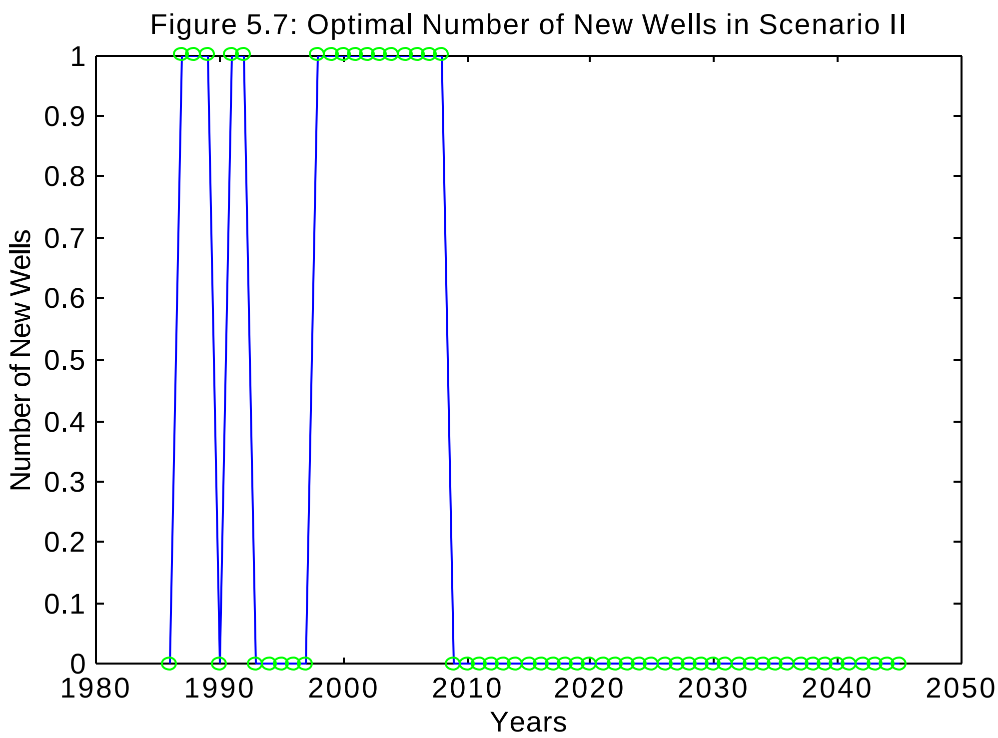


Figure 5.7: Optimal Number of New Wells in Scenario II
Number of New Wells
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1980
1990
2000
2010
2020
2030
2040
2050
Years

Figure 5.8: Optimal Extraction Rates in Scenario III

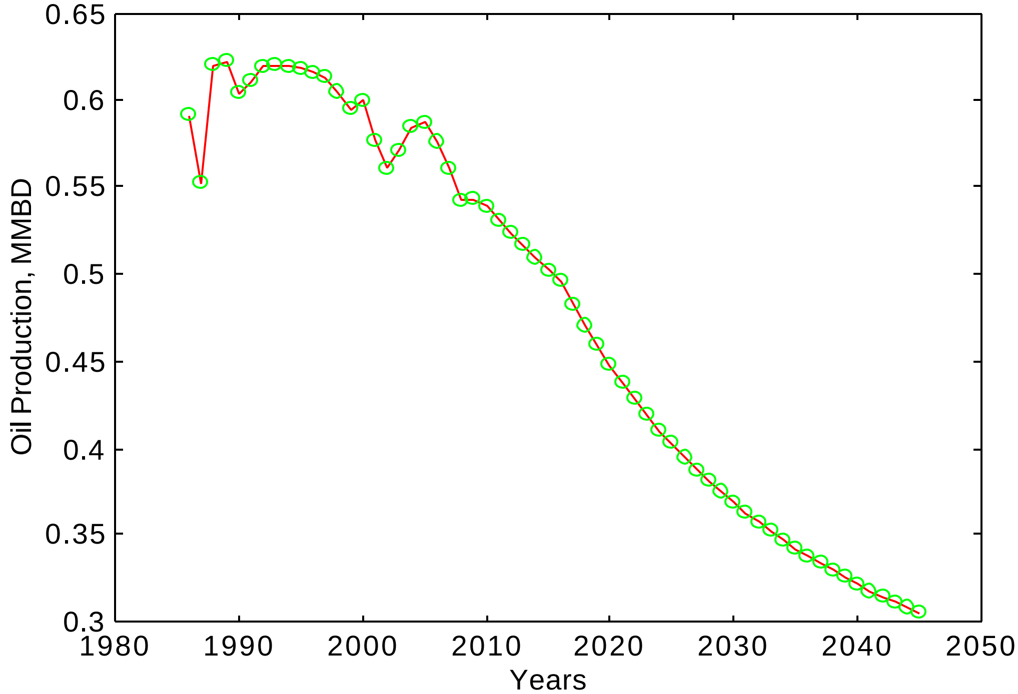


Figure 5.9: Optimal Number of New Wells in Scenario III

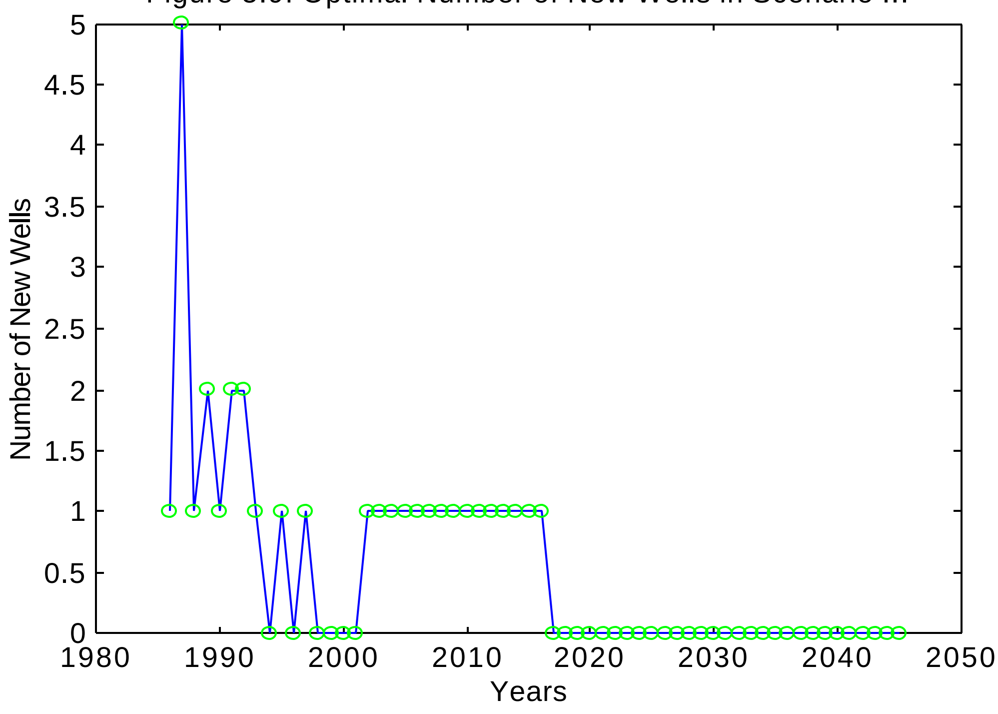


Figure 5.10: Optimal Extraction Rates in Scenario IV

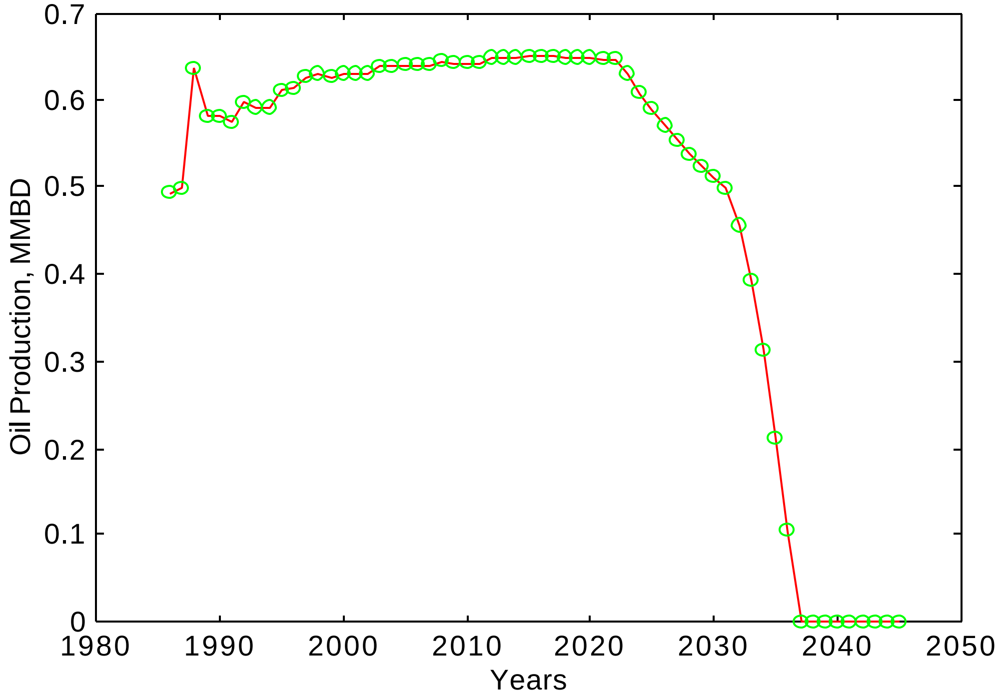


Figure 5.11: Optimal Number of New Wells in Scenario IV

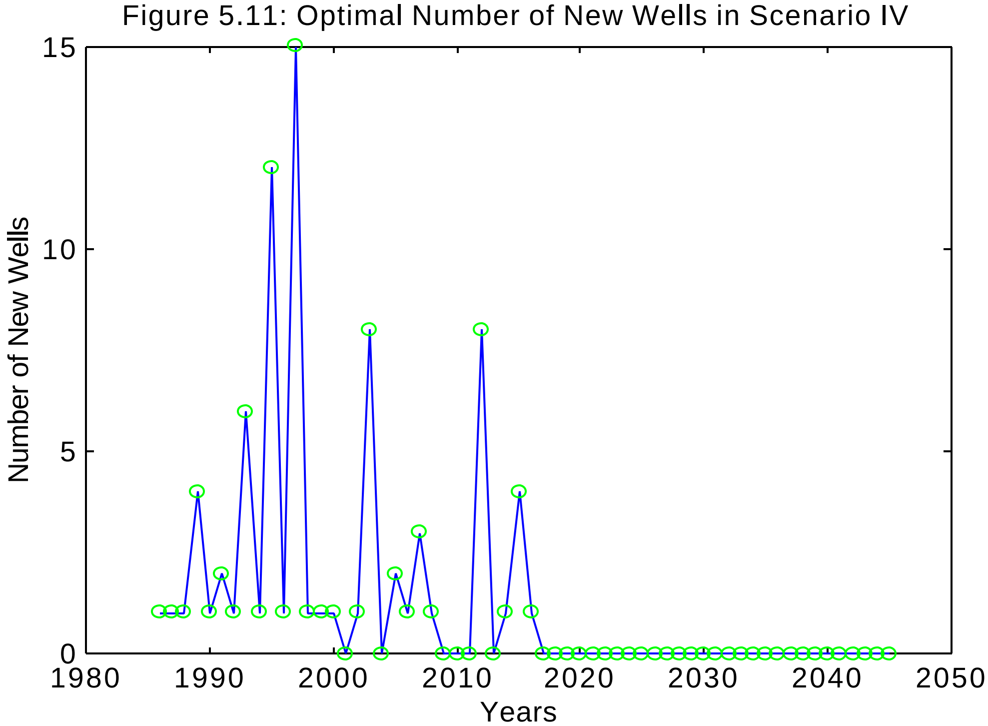


Figure 5.12: A Comparison of Simulated Optimal Productions with the Actual Path

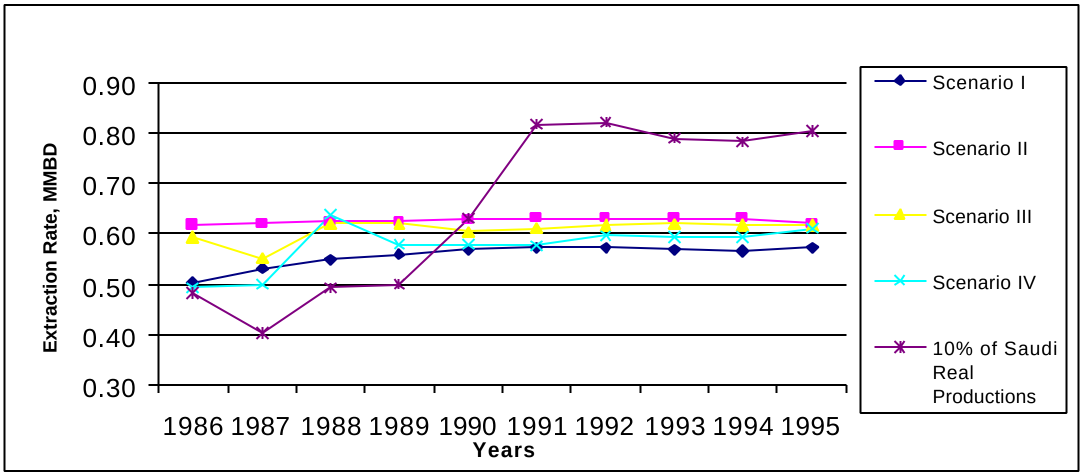
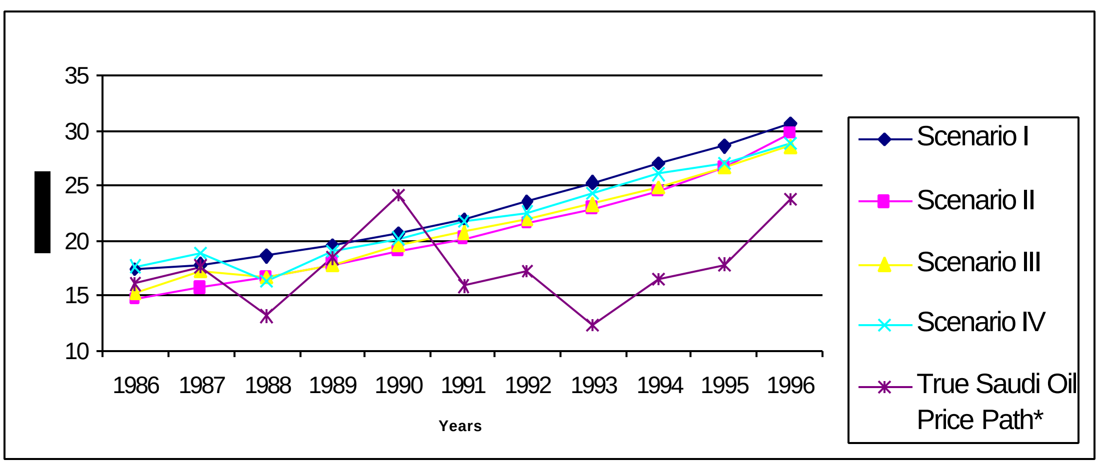


Figure 5.13: A Comparison of Simulated Price Path in Each Scenario
with the Actual Saudi Oil Price Path



- Real Price Data Source: *Energy Statistics Sourcebook*, PennWell Publishing Company, p. 438, 12th ed., 1997

Figure A1

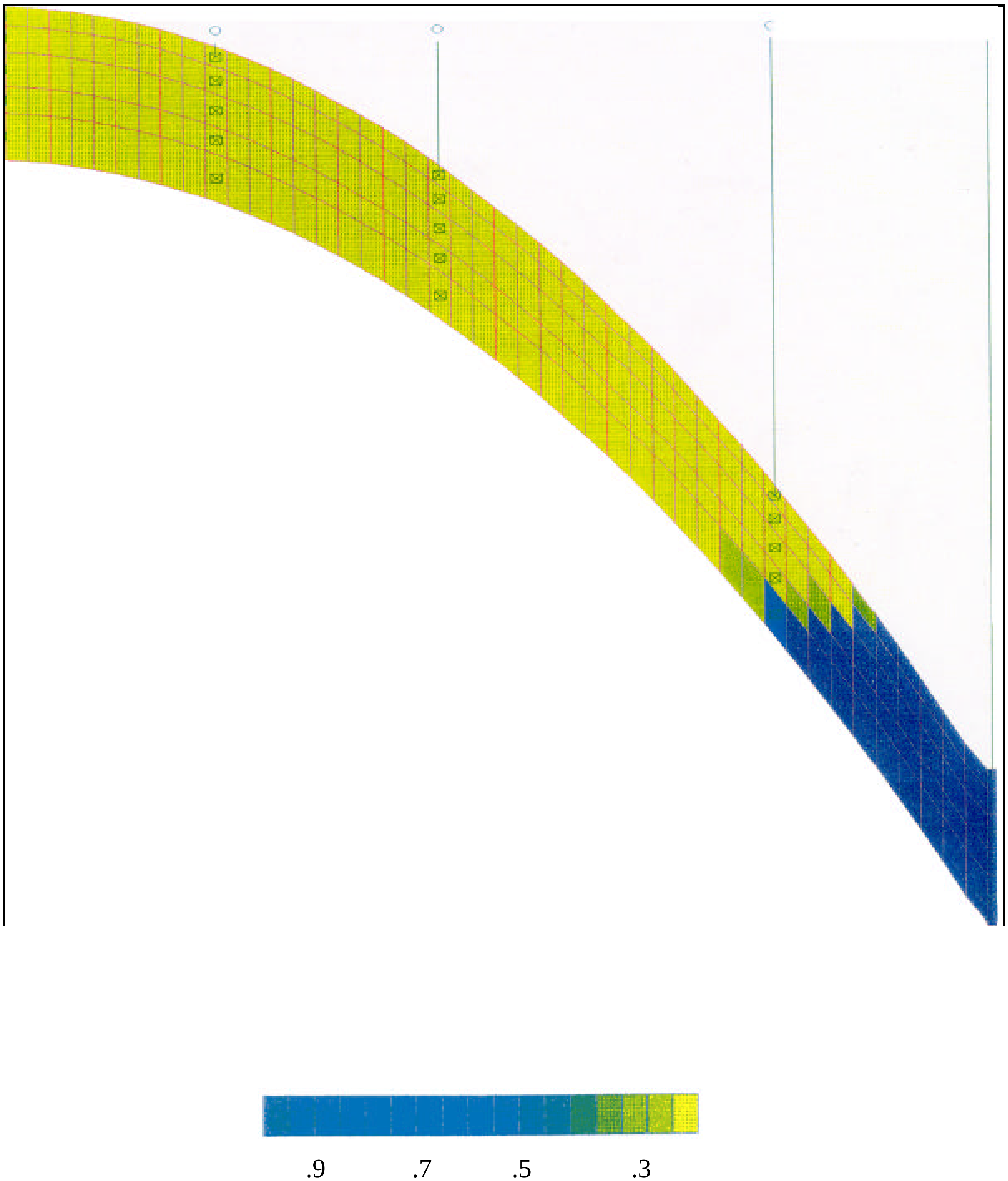


Figure A2

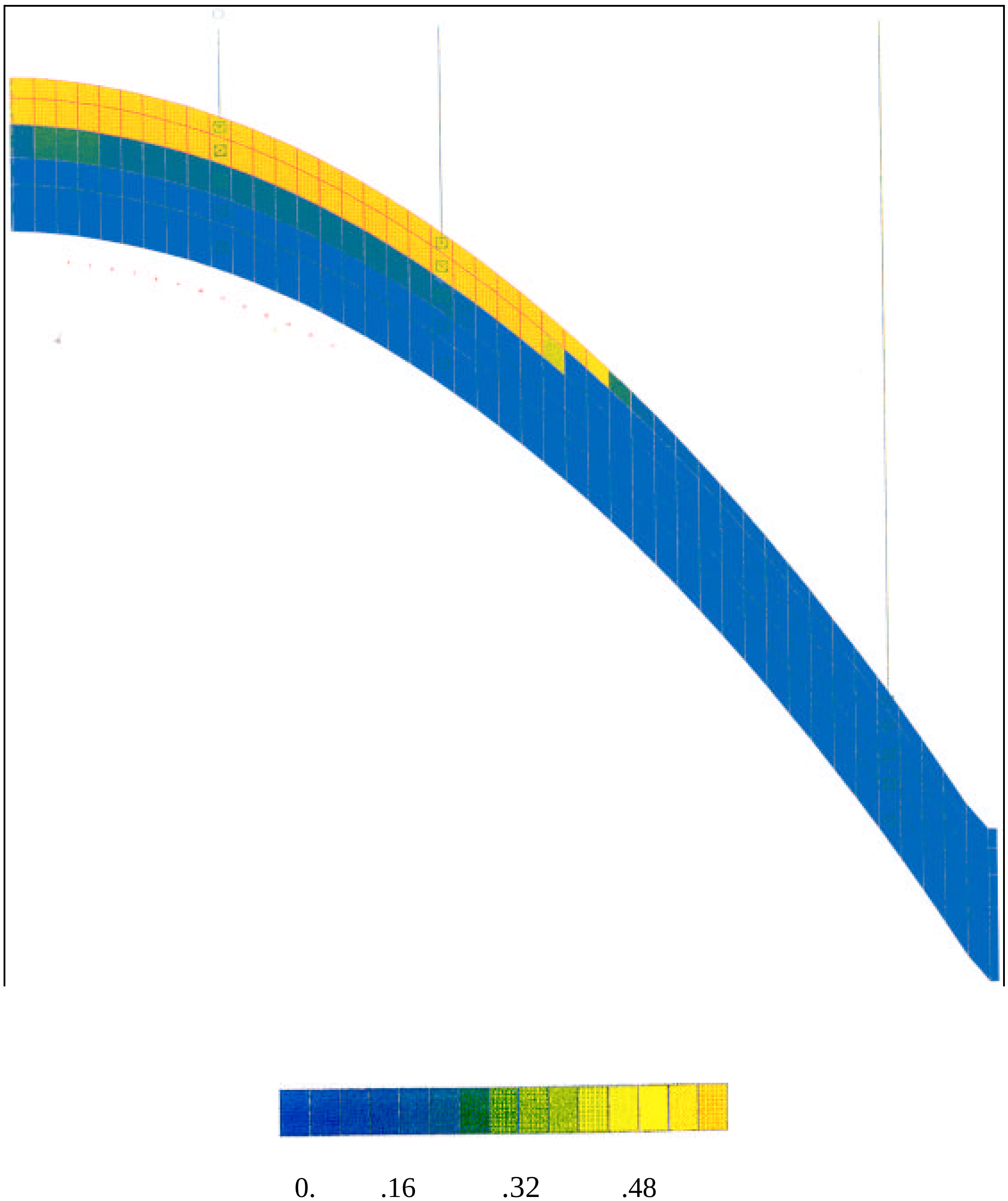


Figure A3

Figure A4

Typical Well Production Schedule

(Rates at 5% of Reserves Per Year)