Phase Structure of Driven Quantum Systems

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Periodically driven ("Floquet") systems are considered as promising candidates for exhibiting orders in analogous to those traditionally studied in equilibrium statistical mechanics. Though in the clean and non-interacting limit, Floquet systems are believed to only host the infinite temperature ergodic phase, Floquet many-body localization induced by disorder could stabilize the nontrivial phases. Here we show that, under the notion of eigensystem order, binary driven one dimensional spin chains display paramagnetic (PM) and spin-glass (SG) phases, which are defined in parallel with their equilibrium counterparts. Moreover, two phases that are completely new to driven systems are spotted: the 0π phase and the π-SG phase. The results shed light on the rich phase structure of periodically driven systems and open up a new area of research in both statistical mechanics and condensed matter physics.

I. INTRODUCTION

The idea of extending the notion of phase and phase transition to closed/isolated quantum systems away from equilibrium is of great interest in statistical quantum mechanics and condensed matter physics. Remarkable progress has been made both experimentally, especially in cold atom systems[2], and theoretically from various aspects, including the adoption of the well-established random matrix theory for disordered many-body systems[7,9]. Recently this idea has been thoroughly explored in periodically driven quantum systems, namely Floquet systems[8,10], whose Hamiltonian is time-dependent and satisfies $H(t + T) = H(t)$. Interestingly, though for generic Floquet systems, thermodynamics predicts an entropy maximizing state at late times where all static and dynamic correlations become trivial and independent of starting state, a recent work by Khemani et al.[5] demonstrates that, introducing disorders can localize spatial modes and enable rich phase structures, some of which are novel to driven systems and have no equilibrium counterpart.

A. Thermalization and Localization

Thermalization in a closed quantum system is different from an usual system with an external bath. While the entire system experiencing unitary time evolution, for a local subsystem, the rest of the system serves as an internal bath and eliminate local memory at late times, bringing it to thermal equilibrium. Formally,

$$\lim_{t\to\infty}\lim_{V\to\infty} \mathrm{Tr}[O\rho(t)] = \lim_{V\to\infty}[O\rho_{eq}(\beta,\mu,...)], \qquad (1)$$

where ρ_{eq} is the density matrix for the equilibrium ensemble dependent on thermodynamic parameters (β, μ) set by conserved densities in the initial state. The celebrated notion of *eigenstate thermalization hypothesis* (ETH)[3] ensures that eigenstate expected values agree with the expected thermodynamic ensemble averages. In contrast, many-body localization (MBL)[1,4] systems do not reach such thermal equilibrium and local properties fluctuate strongly between states at the same energy density.

B. Floquet Basics

The dynamics of Floquest systems are governed by the Floquet unitary $U_F \equiv U(T)$, which is the time evolution operator over one period:

$$U_F = U(T) = \mathcal{T} e^{-i\int_0^T dt' H(t')}. \qquad (2)$$

The Floquet Hamiltonian is defined via $U_F = e^{-iH_F}$, which renders the Hamiltonian to be stroboscopically time-independent on the eigenstate basis of H_F: $H_F|\nu\rangle = E_\nu|\nu\rangle$. E_ν is defined mod $2\pi/T$ and is thus denoted as quasi-energy to distinguish from the regular conserved energy. The Floquet version of ETH asserts that each Floquet eigenstate thermalize to infinite temperature $\beta = 0$. However, it has been shown that localization can prevent such indefinite heating in many-body interacting systems. Consider a system with an external drive $V\cos(\omega t)$ and a local disorder scale $\sim W$. For weak enough amplitude and $\omega \gg W$, the drive could not generate resonance between localized modes and thus the heating process is avoided.

C. Eigensystem Order

In conventional studies of phases and orders, the order parameters, which serve as diagnosis of ordered phases, are evaluated in equilibrium Gibbs ensembles. Such idea need to be generalized for out-of-equilibrium orders. Instead of taking thermal averages, order parameters and correlation functions are calculated on single eigenstates. In addition, properties of eigenspectrum, e.g. level spacing statistics, can be used to characterize static or dynamical nature of system.

II. MODELS AND RESULTS

A. One Dimensional Spin Chain

We start from considering an undriven 1D disordered spin chain with Ising Symmetry,

$$H = \sum_i J_i \sigma_i^x \sigma_{i+!}^x + \sum_i h_i \sigma_i^z + J_z \sum_i \sigma_i^z \sigma_{i+1}^z \tag{3}$$

Carrying out a Jordan-Wigner transformation, the Hamiltonian is mapped to a fermionic picture. The first two terms give a p-wave superconducting free-fermion model, whereas the final term is converted to density-density interactions. In the non-interacting clean model limit, $J_i = J$, $h_i = h$ and $J_z = 0$, the system holds two ground states: a paramagnetic (PM) state with spins aligned with the external field when $J < h$ and a ferromagnetic state with the Ising symmetry spontaneously broken when $J > h$. At any finite system size, the two lowest-lying eigenstates for the ferromagnet are Z_2 symmetric "Schrodinger cat states": $|0_\pm\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle \pm |\downarrow\rangle]$. The characteristic difference between the two states is that the latter displays *long-range order* (LRO), where the correlator $C(i,j) = \langle \sigma_i^z \sigma_j^z \rangle - \langle \sigma_i^z \rangle \langle \sigma_j^z \rangle$ remains finite upon taking long distance and infinite system size limit.

Introducing disorder to the system changes the picture described above. When $h = J_z = 0$ and J_i are randomly distributed, each eigenstate becomes "glassy": the Ising symmetry is only locally broken and is energetically degenerate with its Ising reversed partner. Turning on weak fields, the finite size Z_2 eigenstates are again Schrodinger cat states. This glassy phase is diagnosed by two point correlation functions evaluated in each Schrodinger cat eigenstate. Note that the sign of such correlation functions fluctuates between different states and the thermal average is cancelled out. Thus distinct from the aforementioned ferromagnetic phase, this phase is denoted as spin-glass (SG). Specifically, we consider log-normally distributed J_i and h_i, with mean $\overline{\log(J_i)} = \overline{\log(J)}$, $\overline{\log(h_i)} = \overline{\log(h)} = 0$ and standard deviation 1. The work on eigenstate order has shown that, with disorder and localization, both the PM phase for $\overline{\log J} < \overline{\log h}$ and the SG phase for $\overline{\log J} > \overline{\log h}$ exist at all energies[5]. With weak interactions, $0 < J_z \ll 1$, the strongly localized PM and SG phases remain MBL.

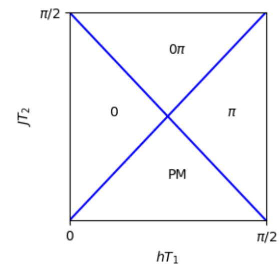

FIG. 1: Phase diagram of the binary driven spin chain in the non-interacting limit. Figure adapted from "Phase Structure of Driven Quantum Systems", by Khemani, Vedika and Lazarides, Achilleas and Moessner, Roderich and Sondhi, S. L., Phys. Rev. Lett., 10.1103/PhysRevLett.116.250401[5].

B. Spin Chain with Periodic Binary Drives

Consider a binary drive with alternating mean of $\log J$:

$$H(t) = \sum_i f_s(t) J_i \sigma_i^x \sigma_{i+1}^x + \sum_i h_i \sigma_i^z + J_z \sum_i \sigma_i^z \sigma_{i+1}^z, \tag{4}$$

where $f_s(t) = 1$ when $0 \le t < T/4$ and $3T/4 < t \le T$ and $f_s(t) = e$ when $T/4 \le t \le 3T/4$. We set $J_z = 0.1$. According to the previous discussion, Floquet MBL requires small interactions and large driven frequency compared with the disorder scale. Thus the frequency is set at $\omega = 2\pi/T = 2W$, where W is the standard deviation of h_i and J_i. In order to examine whether the system reaches MBL within the parameter space studied, the level-statistics ratio $r = \min(\delta_n, \delta_{n+1})/\max(\delta_n, \delta_{n+1})$ is calculated for the quasi-energy gaps $\delta_n = E_{n+1} - E_n$. Away from the critical value $\overline{\log(J)} = \overline{\log(h)}$, the disorder averaged $\langle r \rangle$ approaches the Poisson limit of 0.38 with increasing size, which indicates that the system is many-body localized. $\langle r \rangle$ is peaked at $\overline{\log(J)} = \overline{\log(h)}$, nevertheless the value at that point remains below the Circular Orthogonal Ensemble (COE) value of 0.527, showing partial delocalization.

With localization established, the notion of phases can be discussed in a similar manner as in equilibrium systems. Based on the discussion of the on dimensional spin chain, the dynamical counterpart of PM and SG are expected in the driven spin chain. In this case, the diagnostic values are evaluated in Floquet eigenstates instead of Gibbs ensembles. Consider the spin correlators ($A = x$ or y)

$$C_{AA}^{\alpha}(ij;t) = \langle \phi_\alpha(t) | \sigma_i^A \sigma_j^A | \phi_\alpha(t) \rangle \tag{5}$$

for $i - j \gg 1$ and the SG diagnostic χ^{SG} defined as

$$\chi_\alpha^{SG}(t) = \frac{1}{L^2} \sum_{i,j=1}^{L} |\langle \phi_\alpha(t) | \sigma_i^x \sigma_j^x | \phi_\alpha(t) \rangle|^2 \tag{6}$$

in any given Floquet eigenstate. For $\overline{\log}(J) < \overline{\log}(h)$, the correlators C_{xx} and C_{yy} both vanish with increasing system size L and χ^{SG} approaches 0, signaling a PM phase. For $\overline{\log}(J) > \overline{\log}(h)$, both correlators are generically nonzero with varying signs and χ^{SG} is finite for increasing L, showing typical features of a SG phase.

Note that in Floquet systems, it is sufficient to identifying the PM phase, since the SG phase can be obtained by duality[5]. In the localized SG phase, the MB eigenstates come in almost degenerate spin-flip pairs. They can be connected by any spin operators that flips the parity of the eigenstates. Thus the spectral function of the spin raising operator σ_i^+ in the Floquet eigenbasis

$$A(\omega) = \frac{1}{2^L}\sum_{\alpha\beta}\langle\phi_\alpha(0)|\sigma_i^+|\phi_\beta(0)\rangle\delta(\omega-(E_\alpha-E_\beta)) \quad (7)$$

is a delta function peaked at $\omega = 0$ (labeled as the "0" phase).

Moving beyond generalization of phases already exist in equilibrium systems, two new Ising phases are identified: the π-SG phase and the 0π-PM phase. Instead of energetically degenerate pairs, the MB Floquet eigenstates come in cat pairs separated by quasi energy π/T, where the spectral function $A(\omega)$ shows a delta function peak at $\omega = \pi/T$. Moreover, the magnitude of C_{xx} and C_{yy} correlators cross twice during a period, meaning the SG order parameter rotating by an angel π about the z axis. Hence we have introduced four Floquet phases: the long range ordered 0-SG and π-SG phases, and their dualities without long range order, namely the PM and the 0π phases.

Consider the following binary periodic drive: $H(t) = H_z$ if $0 \le t < T_1$ and $H(t) = H_x$ if $T_1 \le t < T = T_1 + T_2$, where

$$H_z = \sum_i h_i\sigma_i^z + J_z\sum_i \sigma_i^z\sigma_{i+1}^z \quad (8)$$

$$H_x = \sum_i J_i\sigma_i^x\sigma_{i+!}^x + J_z\sum_i \sigma_i^z\sigma_{i+1}^z \quad (9)$$

The phase diagram of the Hamiltonian in the non-interacting limit ($J_z = 0$) is plotted in Fig. (1). In order to examine the novel Floquet π-SG phase in an interacting model, T_1 is picked to be 1, T_2 is picked to be $\pi/2$. h_iT_1 is uniformly drawn from the interval $(1.512, 1.551)$ and J_iT_2 from $(0.393, 1.492)$. According to Fig. (1), this choice of parameters falls into the π-SG region. The stability of MBL against interaction is again examined by calculating the level statistics ratio $\langle r\rangle$ as before. A clear transition happens at about $J_z = 0.1$. When the J_z exceeds 0.1, the MBL state is destroyed.

There are two characteristic features of the π-SG phase: (i) The spectral functions measured in the parameter regime of the π-SG phase exhibit clear peak at $\omega = \pi/T$, which gradually disappears as J_z goes beyond the MBL limit. In contrast, the peak of the spectral functions of the 0-SG phase are located at $\omega = 0$. (ii) The time evolution of the correlators C_{xx} and C_{yy} cross each other twice during one period, meaning the SG order parameter rotating by an angle π about the z axis.

III. CONCLUSION

To summarize, contrary to naive expectation, numerical evidence provided above illustrates that Floquet systems host a rich phase diagram beyond the trivial ergodic phase. In addition to Floquet counterparts of the established phases in equilibrium system, there are several phases that are entirely new to nonequilibrium systems. Some of those phases are related to more novel concepts, e.g. time crystal[6]. The work opens up a zoo of new phenomena in Floquet systems awaiting for further exploration.

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